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A Crack Kinked at the Interface of Bonded Anisotropic Elastic Media under Longitudinal Shear Stresses

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The stress intensity factors at the tips of a crack kinked at the interface of bonded anisotropic elastic media are analyzed. By using a singular point method, the problem is reduced to solving a system of singular integral equations with generalized Cauchy kernels. The characteristic equation that determines the stress singularity at the kink vertex is derived from the singular integral equations, and singular stress fields at the kink vertex are determined definitely. By taking into account of the stress singularity at the kink vertex, numerical results are given graphically for the stress intensity factors at the tips of the kinked crack.

Key Words : Kinked crack, Longitudinal shear, Stress intensity factor, Bonded elastic medium, Anisotropic medium, Singular integral equation, Generalized Cauchy kernel.

1. Introduction

The problem of kinked crack, also referred to as the problem of branched crack, is of great importance for brittle materials such as glass, ceramics and polymers. In particular, to achieve high fracture toughness in many brittle composite materials, consideration of cracks kinked at interfaces in bonded materials are essential. A main object of the problem of kinked crack problem is to calculate the stress intensity factors at the tips of the kinked cracks and to determine a criterion for describing the direction of crack initiation and propagation. Therefore, a large number of studies dealing with the problem of kinked crack problem has been made so far, especially for isotropic media [1-10]. Erdogan and his coworkers [1-3] have studied, by using Mellin transform method, the behavior of a crack kinking on the interface at right angle. Similar problems involving penetration and deflection of a main crack terminating at the interface at right angle were also studied [4,5].

The kinked interfacial cracks with oblique angles were first considered by Hayashi and Nemat-Nasser [6] by using the Muskhelishvili's complex potential method and replacing cracks with continuous distributions of dislocations. Later on, the cases of semi-infinite crack [7,8] and finite length crack [9] have also been studied by using the Muskhelishvili's complex potential method. As another work along the same line, a longitudinal shear problem has been analyzed in [10] by employing the Wiener-Hopf technique, which pays special attention to the kinked part of finite length approaching zero.

However, all of those mentioned above are concerned with isotropic elastic media. Despite the significance of anisotropic elastic media in modern industry, the problem of kinked crack in anisotropic elastic media has received much less attention. The studies on the energy release rate [11,12] and the stress intensity factors [13] of the kinked cracks in homogeneous anisotropic elastic media have only been made. In [11-13], a method similar to that of Lo [14] for a infinitesimally small crack in homogeneous isotropic elastic media was employed. In these studies, however, stress singularity at kink vertex has not been considered.

Recently, taking account of the stress singularities at both of the crack tips and the kink vertex, Blanco et. al. [15] analyzed a kinked crack in homogeneous anisotropic elastic media by means of a singular point method.

In the present paper, on the basis of the method in the previous work [16], a crack kinked at the interface of bonded anisotropic elastic media under longitudinal shear stresses is analyzed. Because the fundamental potential functions of the present problem are distinct from that of [16], a different analysis is needed in the present paper. The kinked crack is located in a semi-infinite space of the bonded anisotropic elastic media, and in an arbitrary position and direction. The problem is reduced to solving a system of singular integral equations with generalized Cauchy kernels. By the use of the function-theoretic method [17], a characteristic equation that determines stress singularities at the kink vertex is derived from the singular integral equations. Thus the singular stress fields at the kink vertex are determined definitely. By carrying out numerical calculations, the results of the stress intensity factors at the crack tips were given for various crack geometries as well as the combination of elastic constants of the bonded anisotropic elastic media.

2. Statement of Problem and Basic Equations

A kinked crack in bonded anisotropic elastic media is shown in Fig. 1. The medium is made of two homogeneous semi-infinite spaces I and II with different elastic constants. These two semi-infinite spaces are perfectly bonded together along the common surface L. A rectangular coordinate system (x, y) is located on the interface of the media. The crack lies in the lower semi-infinite space I and kinked at the interface. The subsidiary rectangular coordinate systems $(x_\lambda, y_\lambda)(\lambda=1,2)$ are also located on the crack L_1 of length $2a_1$ and the crack L_2 of length $2a_2$ as shown in Fig. 1. The crack L_1 and the crack L_2 make an angle $\theta_\lambda(\lambda=1,2)$ with the x-axis. The distance between the center of the crack L_1 and the interface is denoted by h_1 and that of the crack L_2 to the interface is h_2. The bonded anisotropic elastic media are subjected to longitudinal shear stresses $\tau_k(k=0,1,2)$ at infinity.

Now, we define the three complex variables referred to the coordinates (x, y) and $(x_\lambda, y_\lambda)(\lambda=1,2)$ as follows :

$$z=x+iy, \quad z_\lambda=x_\lambda+iy_\lambda \quad (\lambda=1,2) \tag{1ab}$$

In the following analysis, we employ the subscript λ for the quantities reterred to the rectangular coordinate systems (x_λ, y_λ) $(\lambda=1,2)$, unless stated otherwise. The indices I and II are used to denote the quantities associated with the lower and upper semi-infinite spaces of the bonded anisotropic elastic media, respectively. We define the complex variables $\zeta_m(m=I,II)$, η_λ $(\lambda=1,2)$ and complex constants $d_m^\lambda(m=I, II; \lambda=1,2)$ as follows :

$$\zeta_m=(z+\gamma_m\bar{z})/(1+\gamma_m) \quad (m=I,II),$$
$$\eta_\lambda(z_\lambda+\Gamma_\lambda z_\lambda/(1+\Gamma_\lambda) \quad (\lambda=1,2) \tag{2}$$
$$d_m^\lambda = ih_1(1-\gamma_m)/(1+\gamma_m) \quad (m=I,II),$$
$$d_2^\lambda = w_2 - ih_2(1-\gamma_m)/(1+\gamma_m) \quad (m=I,II) \tag{3}$$

where the over bar indicates the conjugate complex number.

The relation between ζ_m and η_λ in Eq.(2) is

$$\zeta_m - d_\lambda^m = \eta_\lambda e^{i\alpha_\lambda}(1+\Gamma_\lambda)/(1+\gamma_m) \quad (m=I,II; \lambda=1,2) \tag{4}$$

where γ_m and Γ_λ are the characteristic values referred to the rectangular coordinate systems (x, y) and (x_λ, y_λ), respectively, and the following condition holds :

$$\Gamma_\lambda=\gamma_m e^{-2i\alpha_\lambda} \quad (m=I,II; \lambda=1,2) \tag{5}$$

The value of γ_m which satisfies the condition $|\gamma_m|<1$ is obtained by solving the following algebraic equation :

$$k_{22}^m\gamma^2+2k_{12}^m\gamma+k_{11}^m=0 \quad (m=I,II) \tag{6}$$

where

$$k_{11}^m=C_{55}^m-C_{44}^m+2iC_{45}^m=k_{11}^m, \; k_{12}^m=C_{55}^m+C_{44}^m \quad (m=I,II) \tag{7}$$

and $C_{ij}^m(m=I,II; i,j=4,5)$ are the elastic constants. When the anisotropy is characterized by the elastic symmetry with respect to the x, y plane, the generalized Hooke's law in the condition of longitudinal shear is

$$\sigma_{zy}^m=C_{44}^m\partial u^m/\partial y+C_{45}^m\partial u^m/\partial x,$$
$$\sigma_{zx}^m=C_{45}^m\partial u^m/\partial y+C_{55}^m\partial u^m/\partial x \quad (m=I,II) \tag{8}$$

The stress components σ_{zx}^m and σ_{zy}^m and the displacement u^m in the space m (m = I,II) referring to the rectangular coordinate system (x, y) are expressed by the complex potentials ϕ_m :

$$\sigma_{zy}^m+i\sigma_{zy}^m=k_1^m/(1+\gamma_m)\phi_m'(\zeta_m)+\bar{k}_2^m/(1+\gamma_m)\phi_m'(\zeta_m) \quad (m=I,II) \tag{9}$$
$$\partial u^m/\partial x=\phi_m'(\zeta_m)+\overline{\phi_m'(\zeta_m)} \quad (m=I,II) \tag{10}$$

where

$$k_1^m=k_{11}^m+\gamma_m k_{12}^m, \; k_2^m=k_{12}^m+\gamma_m k_{11}^m \quad (m=I,II) \tag{11}$$

If we denote the complex potential $\Phi_{m\lambda}(\eta_m)$ referred to the rectangular coordinate systems (x_λ, y_λ) in the lower semi-infinite space, the relations between $\phi_I(\zeta_I)$ and $\Phi_{I\lambda}(\eta_\lambda)$ $(\lambda=1,2)$ are

$$\Phi_{I\lambda}'(\eta_1)=e^{i\alpha_1}\{(1+\Gamma_1)/(1+\gamma_I)\}\phi_I'(\zeta_I),$$
$$\Phi_{I\lambda}'(\eta_2)=e^{-i\alpha_2}\{(1+\Gamma_2)/(1+\gamma_I)\}\phi_I'(\zeta_I) \tag{12}$$

where we wrote down to be restricted to only the potentials in the lower semi-infinite space. The relation between ζ_m and η_λ is given by Eq.(4).

The geometry of crack configuration yields :

$$w_2=a_1\cos\alpha_1+a_2\cos\alpha_2, \; h_\lambda=a_\lambda\sin\alpha_\lambda(\lambda=1,2) \tag{13}$$

Moreover, the constants k_λ^m in Eq. (11) are also expressed by

$$k_1^m=K_1^m e^{2i\alpha_1}, \; k_2^m=K_2^m \quad (m=I,II) \tag{14}$$

where we see that k_2^m are the invariant constants, and K_λ^m the constants with respect to the rectangular coordinate systems (x_λ, y_λ).

If the longitudinal shear stresses $\tau_k(k=0,1,2)$ act at infinity of the bonded anisotropic elastic media, the boundary conditions can be expressed as follows :

(a) At infinity $(|\zeta_m|\to\infty)$

$$\sigma_{zx}^I=\tau_1, \; \sigma_{zx}^{II}=\tau_2, \; \sigma_{zy}^I=\sigma_{zy}^{II}=\tau_0 \tag{15}$$

(b) Along the interface (y = 0)

$$\sigma_{zy}^I=\sigma_{zy}^{II}, \; u^I=u^{II} \tag{16}$$

(c) Along the cracks L_λ $(y_\lambda=0, |x_\lambda|<a_\alpha)$

$$\sigma_{z_\lambda y_\lambda}^I=0 \tag{17}$$

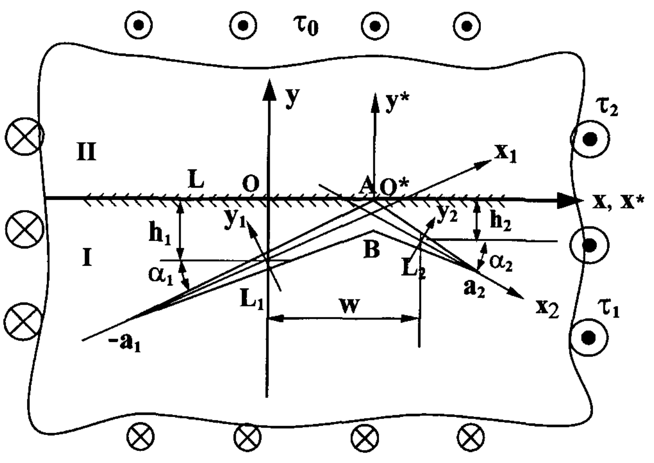

Figure 1 Crack kinked at the interface of bonded anisotropic elastic media and coordinate ststems

3. Singular Integral Equations with Generalized Cauchy Kernels

The cracks L_1 and L_2 under longitudinal shear stresses will be treated as continuous distributions of screw dislocations. By means of this method, a direct determination of the singular stress fields at the crack tips and the kink vertex could be made. To begin with, consider a solution that satisfies the continuity conditions in Eq. (16). The complex potentials for two screw dislocations which locate at $(x^*_{\lambda 0},\ y^*_{\lambda 0})$ in the lower semi-infinite space of the bonded anisotropic elastic media are found by using analytic continuation method as follows :

$$\phi'_{I}(\zeta_{I}) = A_1 b_1/2\pi i \times (\zeta_{I} - \zeta_{I0})^{-1} - A_2 b_1/2\pi i \times (\zeta_{I} - \bar{\zeta}_{I0})^{-1} + A_1 b_2/2\pi i \times (\zeta_{I} - \zeta_{II0})^{-1} - A_2 b_2/2\pi i \times (\zeta_{I} - \bar{\zeta}_{II0})^{-1} \quad (18)$$

$$\phi'_{II}(\zeta_{II}) = A_3/2\pi i \times \left\{ b_1/(\zeta_{II} - \zeta_{I0}) + b_2/(\zeta_{II} - \zeta_{II0}) \right\} \quad (19)$$

where

$$k_m = -(k^m_I - k^m_{II})/\{2(1+\gamma_m)\},\ \zeta^*_m = (z^*_{\lambda 0} + \gamma_m \bar{z}^*_{\lambda 0})/(1+\gamma_m),\ z^*_{\lambda 0} = x^*_{\lambda 0} + iy^*_{\lambda 0} \quad (20)$$

and b_λ $(\lambda = 1,2)$ are the magnitude of the Burgers vector of the screw dislocations, and A_i $(i = 1,2)$ are constants and are given in Appendix A. In the following analysis, we restrict to the orthotropic elastic media, namely $C^m_{45} = 0$. In this case, material constants k_m, k^m_λ and γ_m are real. By using the isolated screw dislocations as the Green's functions, the complex potentials for the crack L_1 and the crack L_2 may be obtained by integration of Eqs.(18) and (19) as follows :

$$\begin{aligned}\Phi'_{I1}(\eta_1) = T^*_1 &+ 1/4\pi i \times \int_{-a_1}^{a_1} b_1(s_1)(\eta_1 - s_1)^{-1} ds_1 \\ &+ B_1/4\pi i \times \int_{-a_1}^{a_1} b_1(s_1)\{s_1 - a_1 - B_2(\eta_1 - a_1)\}^{-1} ds_1 \\ &- B_3/4\pi i \times \int_{-a_2}^{a_2} b_2(s_2)\{s_2 + a_2 - B_4(\eta_1 - a_1)\}^{-1} ds_2 \\ &+ B_5/4\pi i \times \int_{-a_2}^{a_2} b_2(s_2)\{s_2 + a_2 - B_6(\eta_1 - a_1)\}^{-1} ds_2\end{aligned} \quad (21)$$

$$\begin{aligned}\Phi'_{I2}(\eta_2) = T^*_2 &+ 1/4\pi i \times \int_{-a_2}^{a_2} b_2(s_2)(\eta_2 - s_2)^{-1} ds_2 \\ &- C_1/4\pi i \times \int_{-a_1}^{a_1} b_1(s_1)\{s_1 - a_1 - C_2(\eta_2 + a_2)\}^{-1} ds_1 \\ &+ C_3/4\pi i \times \int_{-a_1}^{a_1} b_1(s_1)\{s_1 - a_1 - C_4(\eta_2 + a_2)\}^{-1} ds_1 \\ &+ C_5/4\pi i \times \int_{-a_2}^{a_2} b_2(s_2)\{s_2 + a_2 - C_6(\eta_2 + a_2)\}^{-1} ds_2\end{aligned} \quad (22)$$

where B_i $(i = 1,...,6)$ and C_i $(i = 1,...,6)$ are given in Appendix B, and T^*_1 and T^*_2 correspond to far stress fields of the media given by

$$T^*_1 = \tau_1 e^{i\alpha_1} D_1 + i\tau_0 e^{i\alpha_1} D_2,\ T^*_2 = \tau_1 e^{-i\alpha_2} D_1 + i\tau_0 e^{-i\alpha_2} D_2 \quad (23)$$

where D_i $(i = 1,2)$ are the constants and given in Appendix C. In the derivation of Eqs. (21) and (22), Eqs. (4),(12) and (13) and the following expressions were used :

$$z^*_{10} = s_1 e^{i\alpha_1} - ih_1,\ z^*_{20} = s_2 e^{i\alpha_2} + w_2 - ih_2,$$
$$\tau_1(1+\gamma_1)/(k^I_1 + k^I_2) = \tau_2(1+\gamma_1)/(k^{II}_1 + k^{II}_2) \quad (24)$$

where s_λ are points on the crack lines L_λ. The complex potentials (21) and (22) satisfy the boundary conditions (15) and (16). From the remained stress free boundary condition of Eq.(17), we obtain a system of singular integral equations with generalized Cauchy kernels :

$$\begin{aligned}1/\pi \times \int_{-1}^{1} B_1(S_1)(S_1 - X_1)^{-1} dS_1 &+ 1/\pi \times \int_{-1}^{1} M_{11}(X_1, S_1) B_1(S_1) dS_1 \\ &+ 1/\pi \times \int_{-1}^{1} M_{12}(X_1,S_2) B_2(S_2) dS_2 \\ &+ 1/\pi \times \int_{-1}^{1} M_{13}(X_1,S_2) B_2(S_2) dS_2 = C^*_1\end{aligned} \quad (25)$$

$$\begin{aligned}1/\pi \times \int_{-1}^{1} B_2(S_2)(S_2 - X_2)^{-1} dS_2 &+ 1/\pi \times \int_{-1}^{1} M_{21}(X_2,S_2) B_2(S_2) dS_2 \\ &+ 1/\pi \times \int_{-1}^{1} M_{22}(X_2,S_1) B_1(S_1) dS_1 \\ &+ 1/\pi \times \int_{-1}^{1} M_{23}(X_2,S_1) B_1(S_1) dS_1 = C^*_2\end{aligned} \quad (26)$$

where

$$\begin{aligned}M_{11}(X_1,S_1) &= -1/E_1 \times \mathrm{Re}\left[F_1\{S_1 - 1 - B_2(X_1 - 1)\}^{-1}\right] \\ M_{12}(X_1,S_2) &= 1/E_1 \times \mathrm{Re}\left[F_2\{S_2 + 1 - B_4(X_1 - 1)/R\}^{-1}\right] \\ M_{13}(X_1,S_2) &= 1/E_1 \times \mathrm{Re}\left[F_3\{S_2 + 1 - B_6(X_1 - 1)/R\}^{-1}\right] \\ M_{21}(X_2,S_2) &= -1/E_2 \times \mathrm{Re}\left[F_4\{S_2 + 1 - C_6(X_2 - 1)\}^{-1}\right] \\ M_{22}(X_2,S_1) &= 1/E_2 \times \mathrm{Re}\left[F_5\{S_1 - 1 - C_2 R(X_2 - 1)\}^{-1}\right] \\ M_{23}(X_2,S_1) &= -1/E_2 \times \mathrm{Re}\left[F_6\{S_1 - 1 - C_4 R(X_2 - 1)\}^{-1}\right]\end{aligned} \quad (27)$$

$$C^*_1 = -2\left\{(k^I_1)^2 - (k^I_2)^2\right\}^{-1}/E_1 \times \left[\tau_1\,\mathrm{Im}\left\{(k^I_1 - k^I_2)(k^I_1 e^{-i\alpha_1} - k^I_2 e^{i\alpha_1})\right\} + \tau_0\,\mathrm{Re}\left\{(k^I_1 + k^I_2)(k^I_1 e^{-i\alpha_1} - k^I_2 e^{i\alpha_1})\right\}\right] \quad (28)$$

$$C^*_2 = -2\left\{(k^I_1)^2 - (k^I_2)^2\right\}^{-1}/E_2 \times \left[\tau_1\,\mathrm{Im}\left\{(k^I_1 - k^I_2)(k^I_1 e^{i\alpha_2} - k^I_2 e^{-i\alpha_2})\right\} + \tau_0\,\mathrm{Re}\left\{(k^I_1 + k^I_2)(k^I_1 e^{i\alpha_2} - k^I_2 e^{-i\alpha_2})\right\}\right] \quad (29)$$

and the constants E_i $(i = 1,2)$ and F_i $(i = 1,...,6)$ are given in Appendix D, and the following non-dimensional quantities are introduced :

$$X_j = x_j/a_j,\ S_j = s_j/a_j,\ R = a_2/a_1,\ B_j(S_j) = b_j(s_j),\ (j = 1,2) \quad (30)$$

The single-valuedness of the displacement yields :

$$\int_{-1}^{1} B_1(S_1) dS_1 + R\int_{-1}^{1} B_2(S_2) dS_2 = 0 \quad (31)$$

4. Stress Singularity at the Kink Vertex

The order of stress singularities at the kink vertex which meets at the interface is different from that at the other crack tips. By denoting the order of the stress singularity at the kink vertex by ω, and taking into account that the stress singularity at the crack tips is $-1/2$, we assume the density functions $B_\lambda(S_\lambda)$ as follows :

$$\begin{aligned}B_1(S_1) &= G_1(S_1)(1 - S_1)^{\omega}(1 + S_1)^{-1/2} \\ &= G_1(S_1)e^{-i\omega\pi}(S_1 - 1)^{\omega}(S_1 + 1)^{-1/2} \\ &\qquad \text{for } -1 < \omega < 0,\ -1 \le S_1 \le 1\end{aligned} \quad (32)$$

$$\begin{aligned}B_2(S_2) &= G_2(S_2)(1 - S_2)^{-1/2}(1 + S_2)^{\omega} \\ &= G_2(S_2)e^{-i\pi/2}(S_2 - 1)^{-1/2}(S_2 + 1)^{\omega} \\ &\qquad \text{for } -1 < \omega < 0,\ -1 \le S_2 \le 1\end{aligned} \quad (33)$$

where $G_\lambda(S_\lambda)$ are bounded functions in the closed interval $|S| \le 1$. By substituting Eqs.(32) and (33) into Eqs. (25) and (26), and noting that $G_1(-1)$ and are non-zeros, the following characteristic equation which determines the stress singularity ω is obtained :

$$\begin{aligned}&\left[\left\{E_1 \cos\omega\pi - \mathrm{Re}\left[F_1 e^{-i\omega\pi}(B_2)^{\omega}\right]\right\} \times \left\{-E_2\cos\omega\pi + \mathrm{Re}\left[F_4 e^{i\omega\pi}(C_6)^{\omega}\right]\right\} + \right. \\ &+ \left[-\left\{\mathrm{Re}\left[H_1(B_4)^{\omega}\right] + \mathrm{Re}\left[F_3(B_6)^{\omega}\right]\right\} \right. \\ &\left.\left. \times \left\{\mathrm{Re}\left[H_2(C_2)^{\omega}\right] - \mathrm{Re}\left[F_6(C_4)^{\omega}\right]\right\}\right]\right]\end{aligned} \quad (34)$$

where H_1 and H_2 are the constants and are given in Appendix E. Equation (34) shows that the stress singularity ω depends on the elastic constants as well as the kink angle. It should be noted that Eq.(34) has multiple solutions of order ω, which correspond to those of two wedge problems at the kink points A and B in Fig. 1 where the stresses are generally singular except for particular cases. In the case of nonhomogeneous isotropic elastic media, we have $k_1^I = k_1^{II} = 0$, $k_2^I = 2G_I$, $k_2^{II} = 2G_{II}$, $k_I = G_I$, $k_{II} = G_{II}$, $\gamma_m = 0$ and then Eq.(34) becomes

$$[\cos\omega\pi - (1-\Gamma)/(1+\Gamma)\cos\{\omega\pi - 2\alpha_1(1+\omega)\}]$$
$$[\cos\omega\pi - (1-\Gamma)/(1+\Gamma)\cos\{\omega\pi - 2\alpha_2(1+\omega)\}]$$
$$-[\cos(\alpha_1+\alpha_2)(1+\omega) - (1-\Gamma)/(1+\Gamma)\cos(\alpha_1-\alpha_2)$$
$$(1+\omega)]^2 = 0 \qquad (35)$$

where $\Gamma = G_{II}/G_I$ and G_m (m = I,II) are shear elastic constants. Numerical results of Eq.(34) coincide with those obtained by Mellin transform method [18]. Especially, in the case of $\Gamma = 1$, i.e. the homogeneous isotropic elastic medium, Eg.(35) yields

$$\cos^2\omega\pi - \cos^2(\alpha_1+\alpha_2)(1+\omega) = 0 \qquad (36)$$

5. Singular Stress Fields at the Crack Tips and the Kink Vertex

The singular stress fields at the tips of the kinked crack as well as the kink vertex which meets at the interface will be determined by using the function theoretic method developed by Erdogan [17]. If we emplay the polar coordinate systems as shown in Fig. 2, the singular stress fields at the tip of the kinked crack are determined by taking account of the asymptotic behavior of Cauchy integrals [19] as follows :

$$\sigma_{Iz_1x_1} = 2^\omega\sqrt{a_1}\,G_1(-1)/4\sqrt{r_1} \times \mathrm{Re}\left[(K_1^I - K_2^I)(1+\Gamma_1)^1\mu_1\right.$$
$$\left.(\cos\theta + \mu_1\sin\theta)^{-1/2}\right] \qquad (37)$$

$$\sigma_{Iz_1y_1} = -2^\omega\sqrt{a_1}\,G_1(-1)/4\sqrt{r_1} \times \mathrm{Re}\left[(K_1^I - K_2^I)(1+\Gamma_1)^{-1}\right.$$
$$\left.(\cos\theta + \mu_1\sin\theta)^{-1/2}\right] \qquad (38)$$

where $\mu_1 = i(1-\Gamma_1)/(1+\Gamma_1)$, Γ_1 denotes the characteristic value with respect to the rectangular coordinate system (x_1, y_1) and K_λ^m is given by Eq.(14). Then the stress intensity factor at the tip of the crack L_1 is

$$k_3(-a_1) = \lim\sqrt{2r_1}\left[\sigma_{Iz_1y_1}\right]_{\theta\to 0} = -2^\omega\sqrt{a_1}\,G_1(-1)/2\sqrt{2}$$
$$\times \mathrm{Re}\left[(K_1^I - K_2^I)/(1+\Gamma_1)\right] \qquad (39)$$

Similarly, the singular stress fields at the tip of the crack L_2 are given by

$$\sigma_{Iz_2x_2} = 2^\omega\sqrt{a_2}\,G_1(1)/4\sqrt{r_2} \times \mathrm{Re}\left[(K_1'^I - K_2'^I)(1+\Gamma_2)^{-1}\mu_2\right.$$
$$\left.(\cos\theta + \mu_2\sin\theta)^{-1/2}\right] \qquad (40)$$

$$\sigma_{Iz_2y_2} = -2^\omega\sqrt{a_2}\,G_2(1)/4\sqrt{r_2} \times \mathrm{Re}\left[(K_1'^I - K_2'^I)(1+\Gamma_2)^{-1}\right.$$
$$\left.(\cos\theta + \mu_2\sin\theta)^{-1/2}\right] \qquad (41)$$

where $\mu_2 = i(1-\Gamma_2)/(1+\Gamma_2)$. In Eqs.(40) and (41), Γ_2 and K'^I_λ denote the characteristic value and the constants $k_\lambda^{(m)}$ are referred to the rectangular coordinate system (x_2, y_2), respectively.

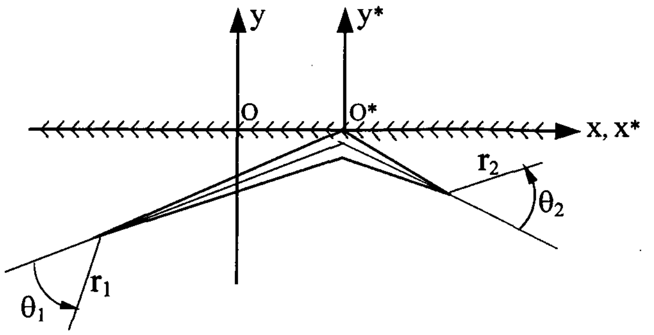

Figure 2 Polar coordinate systems at the crack tips.

Table 1 Elastic constants

	C_{55}	C_{45}	C_{44}
A	1	0	1
B	1	0	2
C	1	0	0.5

Table 2 Combinations of elastic constants for bonded anisotropic elastic Media

Lower semi-infinite space	Upper semi-infinite space	Symbols
A	A	A/A
	B	B/A
	C	C/A

Thus the stress intensity factor at the tip of the crack L_2 is

$$k_3(a_2) = \lim\sqrt{2r}\left[\sigma_{Iz_2y_2}\right]_{\theta\to 0} = -2^\omega\sqrt{a_2}\,G_2(1)/2\sqrt{2}$$
$$\times \mathrm{Re}\left[(K_1'^I - K_2'^I)/(1+\Gamma_2)\right] \qquad (42)$$

Next, we will proceed to determine the interfacial stress fields at the kink vertex. As shown in Fig. 2, a rectangular coordinate system (x^*, y^*) is located at the kink vertex. The complex potential ϕ_{II}^* referring to the rectangular coordinate system (x^*, y^*) in the upper semi-infinite space, which is derived from Eq.(19), is written as

$$\phi^{*\prime}_{II}(\zeta^*_{II}) = T_2^* - A_5/2\pi i \times (1+\gamma_I)(e^{i\alpha_1} + \gamma_I e^{-i\alpha_1})^{-1}$$
$$\times \int_{-a_1}^{a_1} b_1(s_1)\{s_1 - a_1 - (1+\gamma_1)(e^{-i\alpha_1} + \gamma_I e^{-i\alpha_1})^{-1}\zeta^*_{II}\}ds_1$$
$$+ A_5/2\pi i \times (1+\gamma_I)(e^{-i\alpha_2} + \gamma_I e^{i\alpha_2})^{-1}$$
$$\times \int_{-a_1}^{a_1} b_2(s_2)\{s_2 - a_2 - (1+\gamma_1)(e^{-i\alpha_2} + \gamma_I e^{i\alpha_2})^{-1}\zeta^*_{II}\}ds_2 \qquad (43)$$

where

$$\zeta^*_{II} = (z^* + \gamma_{II}\bar{z}^*)/(1+\gamma_{II}),\ z^* = x^* + iy^* \qquad (44)$$

and T_2^* is given by Eq.(23). Substituting Eq.(43) into Eg.(9), using Eg.(12) after some manipulations, we obtain the singular stress fields at the kink vertex in the upper semi-infinite space, as follows :

$$\sigma_{IIn^*x^*} \cong \mathrm{Re}\left[T_2^*(k_1^{II} + k_2^{II})(1+\gamma_{II})^{-1} + i2^{-3/2}\right.$$
$$(k_1^{II} + k_2^{II})k_I(k_I + k_{II})^{-1}(1+\gamma_I)^{\omega+1}(1+\gamma_{II})^{-1}\times$$
$$\times (X^*)^\omega/\sin\omega x \times \left\{G_1(1)(e^{i\alpha_1} + \gamma_{II} e^{-i\alpha_1})^{-\omega-1}\right.$$
$$\left.\left. - G_2(-1)e^{-i\omega\pi}R^{-\omega}(e^{-i\alpha_2} + \gamma_I e^{i\alpha_2})^{-\omega-1}\right\}\right] \qquad (45)$$

$$\sigma_{IIn^*y^*} \cong \mathrm{Im}\left[T_2^*(k_1^{II} - k_2^{II})(1+\gamma_{II})^{-1} + i2^{-3/2}\right.$$
$$(k_1^{II} - k_2^{II})k_I(k_I + k_{II})^{-1}(1+\gamma_I)^{\omega+1}(1+\gamma_{II})^{-1}\times$$
$$\times (X^*)^\omega/\sin\omega x \times \left\{G_1(1)(e^{i\alpha_1} + \gamma_I e^{-i\alpha_1})^{-\omega-1}\right.$$
$$\left.\left. - G_2(-1)e^{-i\omega\pi}R^{-\omega}(e^{-i\alpha_2} + \gamma_I e^{i\alpha_2})^{-\omega-1}\right\}\right] \qquad (46)$$

where

$$X^* = x^*/a_1 \qquad (47)$$

The singular integral equations (25) and (26) and the subsidiary condition (31) can be solved numerically on a straightforward manner. In the present paper, the technique proposed by Erdogan [17] was employed to obtain $G_\lambda(S_\lambda)$ at a set of collocation points in the interval $|S_\lambda| \le 1$.

6. Numerical Results and Discussion

The influences of various factors such as anisotropy of the media, crack configurations and situations on the stress intensity factors at the crack tip are clarified. The elastic constants used in the numerical calculations are given in Table 1, in which the symbol A denotes an isotropic medium, the symbols B and C orthotropic media. Table 2 shows the combinations of those elastic constants. Note that A/A is the case of homogeneous isotropic medium and C/A and B/A are the cases of isotropic medium bonded to an orthotropic medium. Figure 3 shows the variation of the order of stress singularity ω at the kink vertex as a function of kink angle α_2 for $\alpha_1=90°$. The results coincide with those obtained by the different method [18]. In order to check the accuracy of the numerical results, the closed form solution [20] for a special case of the present analysis, i.e. the stress intensity factor for a kinked crack in a homogeneous isotropic elastic medium was used. Comparing the result with that of the closed form solution, we chose the number of collocation points in numerical calculations to hold a difference between them within 1 %. Figure 4 shows an example of the accuracy of the numerical

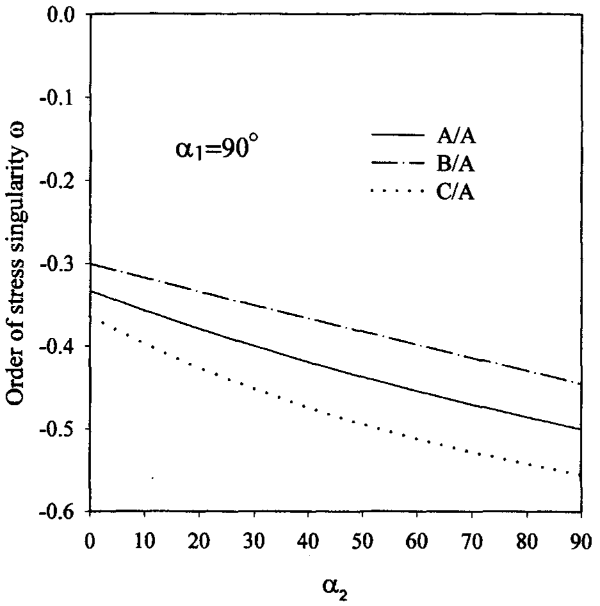

Figure 3 Variation of the order of stress singularity at the kink vertex

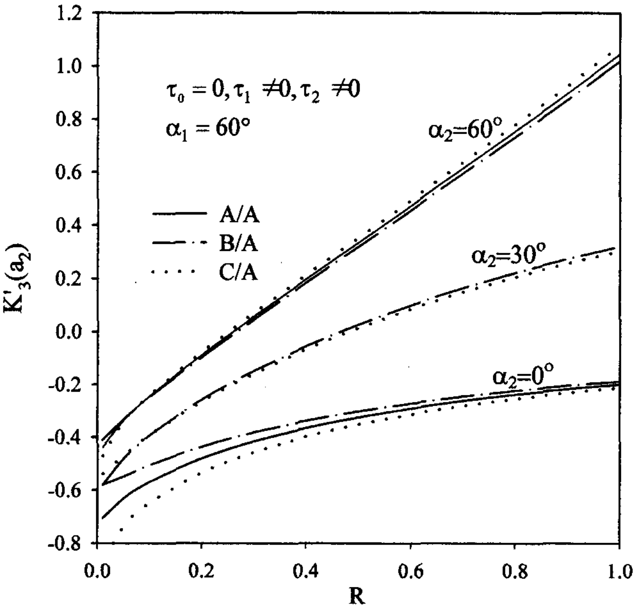

Figure 5-a Influences of anisotropy and R on the non-dimensional stress intensity factor

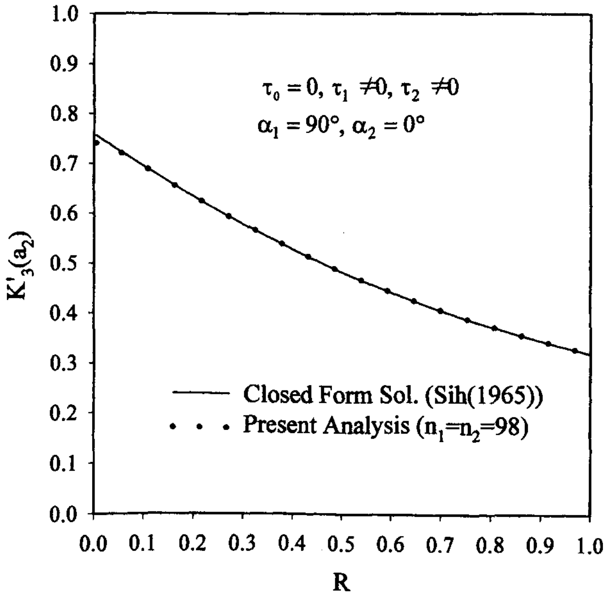

Figure 4 Accuracy of numerical results for a homogeneous isotropic elastic medium

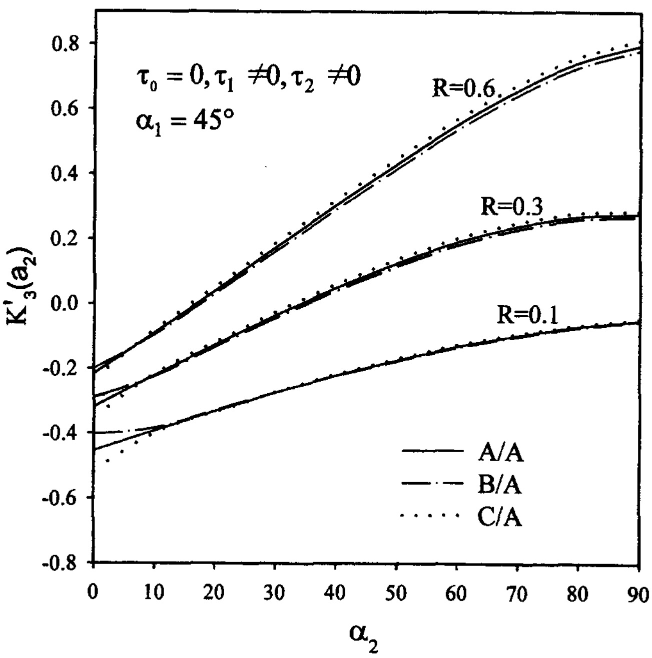

Figure 5-b Influences of anisotropy and α_2 on the non-dimensional stress intensity factor

results for $\tau_0 = 0$, $\tau_1 \neq 0$, $\tau_2 \neq 0$, $\alpha_1 = 90^\circ$ and $\alpha_2 = 0^\circ$, where the non-dimensional stress intensity factor $K'_3(a_2)$ is defined as

$$K'_3(a_2) = k_3(a_2)/k'_3 \tag{48}$$

where

$$k'_3(a_2) = \tau_1\sqrt{a_1 + a_2\cos(\alpha_1 + \alpha_1)} \tag{49}$$

Equation (49) corresponds to the stress intensity factor for the straight crack of length $a_1 + a_2\cos(\alpha_1 + \alpha_2)$ when the stress τ_1 acts at infinity. Figures 5 and 6 show the variation of non-dimensional stress intensity factor $K'_3(a_2)$, $K''_3(a_2)$ at the tip $x_2 = a_2$ versus R and α_2 for the case of the bonded anisotropic elastic media, where non-dimensional stress intensity factor $K''_3(a_2)$ is defined as

Table 3 Numerical values of the order of stress singularity ω at the kink vertex for $\alpha_1 = 90^\circ$

α_2	ω		
	A/A	B/A	C/A
0°	- 0.3333	- 0.3007	- 0.3640
30°	- 0.4000	- 0.3504	- 0.4520
60°	- 0.4545	- 0.3982	- 0.5118
89.99°	- 0.4999	- 0.4450	- 0.5548

Table 4 Numerical values of the non-dimensional stress intensity Factor $K'_3(a_2)$ for homogeneous isotropic elastic medium and $\tau_0 = 0$, $\tau_1 \neq 0$, $\tau_2 \neq 0$, $\alpha_1 = 90^\circ$, $\alpha_2 \neq 0^\circ$

R	Present analysis ($n_1=n_2=98$)	Closed form solution (Sih(1965))
0.2	0.6358	0.6358
0.1	0.6951	0.6958
0.05	0.7257	0.7273

Table 5 Numerical values of the non-dimensional stress intensity factors $K'_3(a_2)$ and $K''_3(a_2)$ for R and α_2 in the case of $\alpha_2 = 60^\circ$

α_2	R	$K_3'(a_2)$ for $\tau_0=0, \tau_1\neq0, \tau_2\neq0, \alpha_1=60°$			$K_3''(a_2)$ for $\tau_0\neq0, \tau_1=0, \tau_2=0, \alpha_1=60°$		
		A/A	B/A	C/A	A/A	B/A	C/A
0°	0.2	- 0.4802	- 0.4361	- 0.4566	0.8165	0.7182	0.9487
	0.1	- 0.5680	- 0.5051	- 0.5341	0.7233	0.6332	0.8437
	0.05	- 0.6298	- 0.5475	- 0.6472	0.6486	0.5621	0.7644
30°	0.2	- 0.2584	- 0.2654	- 0.2484	0.8240	0.8149	0.8325
	0.1	- 0.3930	- 0.3922	- 0.3914	0.7094	0.6996	0.7187
	0.05	- 0.4799	- 0.4688	- 0.4900	0.6202	0.6075	0.6333
60°	0.2	- 0.0886	- 0.0971	- 0.0778	0.7081	0.7076	0.7076
	0.1	- 0.2448	- 0.2478	- 0.2401	0.5971	0.5952	0.5981
	0.05	- 0.3365	- 0.3324	- 0.3400	0.5135	0.5086	0.5181

Table 6 Numerical values of the non-dimensional stress intensity factors $K'_3(a_2)$ and $K''_3(a_2)$ for R and α_2 in the case of $\alpha_2 = 45^\circ$

R	α_2	$K_3'(a_2)$ for $\tau_0=0, \tau_1\neq0, \tau_2\neq0, \alpha_1=45°$			$K_3''(a_2)$ for $\tau_0\neq0, \tau_1=0, \tau_2=0, \alpha_1=45°$		
		A/A	B/A	C/A	A/A	B/A	C/A
0.2	10°	- 0.2924	- 0.2916	- 0.2911	0.9107	0.8857	0.9379
	20°	- 0.2132	- 0.2164	- 0.2078	0.9122	0.8956	0.9291
	30°	- 0.1378	- 0.1433	- 0.1300	0.9016	0.8907	0.9118
0.1	10°	- 0.3917	- 0.3835	- 0.3990	0.8467	0.8202	0.8750
	20°	- 0.3316	- 0.3283	- 0.3333	0.8423	0.8243	0.8604
	30°	- 0.2742	- 0.2740	- 0.2726	0.8277	0.8156	0.8392
0.05	10°	- 0.4646	- 0.4468	- 0.4832	0.7932	0.7621	0.8272
	20°	- 0.4165	- 0.4048	- 0.4279	0.7838	0.7616	0.8071
	30°	- 0.3695	- 0.3619	- 0.3764	0.7659	0.7498	0.7822

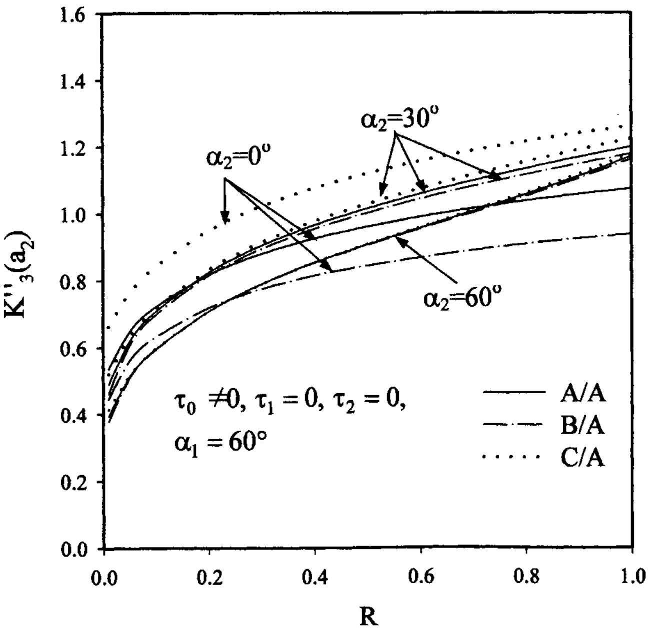

Figure 6 Influences of anisotropy and R on the non-dimensional stress intensity factor $K''_3(a_2)$

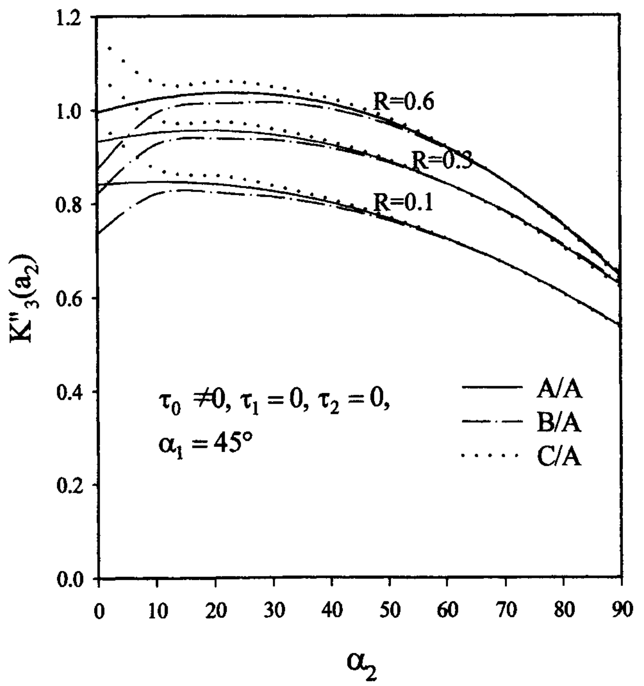

Figure 6 Influences of anisotropy and R on the non-dimensional stress intensity factor $K''_3(a_2)$

$$K''_3(a_2) = k_3(a_2)/k''_3 \qquad (50)$$

where

$$k''_3(a_2) = \tau_0\sqrt{a_1 + a_2\cos(\alpha_1 + \alpha_2)} \qquad (51)$$

In Figs. 5-a,b that shows the results for $\tau_0 = 0$, $\tau_1 \neq 0$, $\tau_2 \neq 0$, it is observed that the influence of anisotropy is remarkable for $\alpha_2 = 0^\circ$, especially smaller values of R. As could be expected from the figures, $K'_3(a_2)$ decreases with decreasing R, and its tendency of decreasing depends on the angle α_2. It is worthwhile to note in Figs. 5-a,b that $K'_3(a_2)$ becomes zero at a certain value of R which is very sensitive to α_2. On the other hand, as can be seen from Figs. 6-a,b, $K''_3(a_2)$ are always positive in the case of $\tau_0 \neq 0$. $\tau_1 = \tau_2 = 0$. The influence of anisotropy is significant for the case of $\alpha_2 = 0^\circ$.

Finally, for the convenience of the readers, numerical values corresponding to Figs. 3 to 6 are shown in Tables 3 to 5.

Acknowledgement

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Appendix A

$$A_1 = k_I/(k_I + k_I),\ A_2 = k_I(k_I - k_{II})/\{(k_I + k_I)(k_I + k_{II})\},$$
$$A_3 = k_I/(k_I + k_{II}) \qquad (a1)$$

Appendix B

$$B_1 = (k_I - k_{II})(1+\Gamma_2)e^{i\alpha_1}/\{(k_I + k_{II})(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})\},$$
$$B_3 = (1+\Gamma_1)e^{i\alpha_1}/(e^{-i\alpha_2} + \gamma_I e^{i\alpha_2}),$$
$$B_4 = (e^{i\alpha_1} + \gamma_I e^{-i\alpha_1})/(e^{-i\alpha_2} + \gamma_I e^{i\alpha_2})$$
$$B_5 = (k_I - k_{II})(1+\Gamma_1)e^{i\alpha_1}/\{(k_I + k_{II})(e^{i\alpha_2} + \gamma_I e^{-i\alpha_2})\},$$
$$B_6 = (e^{i\alpha_1} + \gamma_I e^{-i\alpha_1})/(e^{i\alpha_2} + \gamma_I e^{-i\alpha_2}) \qquad (a2)$$

$$C_1 = (1+\Gamma_2)e^{-i\alpha_2}/(e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}),$$
$$C_2 = (e^{-i\alpha_2} + \gamma_I e^{i\alpha_1})/(e^{i\alpha_1} + \gamma_I e^{-i\alpha_1})$$
$$C_3 = (k_I - k_{II})(1+\Gamma_2)e^{-i\alpha_2}/\{(k_I + k_{II})(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})\},$$
$$C_4 = (e^{-i\alpha_2} + \gamma_I e^{i\alpha_2})/(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})$$
$$C_5 = (k_I - k_{II})(1+\Gamma_2)e^{-i\alpha_2}/\{(k_I + k_{II})(e^{i\alpha_2} + \gamma_I e^{-i\alpha_2})\},$$
$$C_6 = (e^{-i\alpha_2} + \gamma_I e^{i\alpha_2})/(e^{i\alpha_1} + \gamma_I e^{-i\alpha_1}) \qquad (a3)$$

Appendix C

$$D_1 = (1+\Gamma_2)/(k_1^I - k_2^I),\ D_2 = (1+\Gamma_2)/(k_1^I - k_2^I) \qquad (a4)$$

Appendix D

$$E_1 = \mathrm{Re}[(k_1^I e^{-2i\alpha_1} - k_2^I)/2(1 + \gamma_I e^{2i\alpha_2})] \qquad (a5)$$
$$E_2 = \mathrm{Re}[(k_1^I e^{2i\alpha_2} - k_2^I)/2(1 + \gamma_I e^{2i\alpha_2})] \qquad (a6)$$
$$F_1 = (k_I - k_{II})(k_1^I e^{-i\alpha_1} - k_2^I e^{i\alpha_1})/\{2(k_I + k_{II})(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})\} \qquad (a7)$$
$$F_2 = (k_1^I e^{-i\alpha_1} - k_2^I e^{i\alpha_1})/\{2(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})\} \qquad (a8)$$
$$F_3 = (k_I - k_{II})(k_1^I e^{-i\alpha_1} - k_2^I e^{i\alpha_1})/\{2(k_I + k_{II})(e^{i\alpha_2} + \gamma_I e^{-i\alpha_2})\} \qquad (a9)$$
$$F_4 = (k_I - k_{II})(k_1^I e^{i\alpha_2} - k_2^I e^{-i\alpha_2})/\{2(k_I + k_{II})(e^{i\alpha_2} + \gamma_I e^{-i\alpha_2})\} \qquad (a10)$$
$$F_5 = (k_1^I e^{i\alpha_2} - k_2^I e^{-i\alpha_2})/\{2(e^{i\alpha_1} + \gamma_I e^{-i\alpha_1})\} \qquad (a11)$$
$$F_6 = (k_I - k_{II})(k_1^I e^{i\alpha_2} - k_2^I e^{-i\alpha_2})/\{2(k_I + k_{II})(e^{-i\alpha_1} + \gamma_I e^{i\alpha_1})\} \qquad (a12)$$

Appendix E

$$H_1 = (k_1^I e^{i\alpha_1} - k_2^I e^{-i\alpha_1})/\{e^{-i\alpha_2} + \gamma_I e^{-i\alpha_2}\},$$
$$H_2 = (k_1^I e^{i\alpha_2} - k_2^I e^{-i\alpha_2})/\{e^{-i\alpha_1} + \gamma_I e^{-i\alpha_1}\} \qquad (a13)$$