Original

Received January 31, 2002
Accepted for Publication February 27, 2002

Transient Thermoelastic Problem of Angle-Ply Laminated Cylindrical Panel due to Nonuniform Heat Supply

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This paper is concerned with the theoretical treatment of transient thermoelastic problem involving an angle-ply laminated cylindrical panel consisting of an oblique pile of layers having orthotropic material properties due to nonuniform heat supply in the circumferential direction. We obtain the exact solution for the two-dimensional temperature change in a transient state, and thermal stresses of a simple supported cylindrical panel under the state of generalized plane deformation. As an example, numerical calculations are carried out for a 2-layered angle-ply laminated cylindrical panel, which is heated from inner surface. Some numerical results for the temperature change, the displacement and the stresses in a transient state are shown in figures. Furthermore, the influence of the radius ratio on the temperature change, the displacement and the stress distributions are investigated.

Key Words : Thermoelasticity, Angle-Ply Laminate, Cylindrical Panel, Trasient State, Generalized Plane Deformation

1. Introduction

Anisotropic laminated composites have been used in various industrial field as structure materials. When such anisotropic laminated composites are used under high temperature environments, it is known that the material characteristic of the lamina and stacking sequence have a great influence on thermomechanical behaviors such thermal stress and thermal deformation. And one of cause of damage in these laminated composites includes delamination. In order to evaluate this phenomenon, the thermal stress analysis taking into account the transverse stress components is necessary. In addition, a transient thermal stress analysis as well as a steady thermal stress analysis becomes important, because maximum thermal stress distribution occurs in a transient state which lasts from the beginning of the heating to the steady state. Therefore, we recently analyzed exactly the three-dimensional transient thermal stress problem of cross-ply laminated rectangular plate due to partial heating [1] and the transient thermal stress problem of angle-ply laminated strip due to nonuniform heat supply in the width direction [2] taking into account all transverse stress components. However, these studies discuss the problem in rectangular coordinates. On the other hand, an anisotroric cylindrical panel is a typical structure element to produce curved structures such as aircrafts, spacecrafts, pressure vessels and so on. Therefore the thermoelastic problems of anisotropic cylindrical panel with curvature are important as well as those of plate models in the design board. Though there are several exact analysis for the isothermal problems of anisotropic laminated cylindrical panel [3-7], there are only a few exact analyses concerned with the thermoelastic problems of anisotropic laminated cylindrical panel taking into account transverse stress components. For example, Huang and Tauchert treated exactly a cross-ply cylindrical panel [8] and a doubly-curved cross-ply laminate [9] with simply supported edges as a three-dimensional thermoelastic problem. Zenkour et al. [10] analyzed thermoelastic problem of composite laminated cylindrical panel using a refined first-order theory under several boundary conditions. These papers, however, treated only the thermal stress problems under the steady state temperature distribution that is linear with respect to the thickness direction. To the authors knowledge, the exact analysis for a transient thermal stress problem of anisotropic laminated panel under two-dimensional temperature distribution has not been reported.

In the present article, we consider an angle-ply laminated cylindrical panel with simply supported edges due to a nonuniform heat supply in the circumferential direction. We analyze exactly the transient thermal stress and thermal deformation of the laminated cylindrical panel as a generalized plane deformation problem taking into account all transverse stress components. The exact thermoelastic solution obtained in this article, will become effective to verify the accuracy of various laminated shell theories and approximation methods.

2. Analysis

We consider an infinitely long, angle-ply laminated cylindrical panel composed of N layers as shown in Figure 1, the angular length of the side in the circumferential direction of which is denoted by θ_0. The panel's inner and outer radii are designated a

and b, respectively. Throughout this article, the index $i(=1, 2, \cdots, N)$ is associated with the ith layer of a laminated cylindrical panel from the inner side. It is assumed that each layer maintains the orthotropic material properties and the fiber direction in the ith layer is alternated with ply angle ϕ_i to the z axis.

2.1 Heat conduction problem

We assume that the laminated cylindrical panel is initially at zero temperature and is suddenly heated from the inner and outer surfaces by surrounding media with relative heat transfer coefficients h_a and h_b. We denote the temperatures of the surrounding media by the functions $T_a f_a(\theta)$ and $T_b f_b(\theta)$ and assume its end surfaces $(\theta=0,\ \theta_0)$ are held zero temperature. Then the temperature distribution shows a two-dimensional distribution in $r-\theta$ plane, and the transient heat conduction equation for the ith layer and the initial and thermal boundary conditions in dimensionless form are taken in the following forms :

$$\frac{\partial \bar{T}_i}{\partial \tau}=\bar{\kappa}_{ri}\left(\frac{\partial^2 \bar{T}_i}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial \bar{T}_i}{\partial \rho}\right)+\frac{\bar{\kappa}_{\theta i}}{\rho^2}\frac{\partial^2 \bar{T}_i}{\partial \theta^2}\ ;\quad i=1\sim N \tag{1}$$

$$\tau=0\ ;\quad \bar{T}_i=0\ ;\ i=1\sim N \tag{2}$$

$$\rho=\bar{a}\ ;\quad \frac{\partial \bar{T}_1}{\partial \rho}-H_a\bar{T}_1=-H_a\bar{T}_a f_a(\theta) \tag{3}$$

$$\rho=R_i\ ;\quad \bar{T}_i=\bar{T}_{i+1}\quad ;\ i=1\sim(N-1) \tag{4}$$

$$\rho=R_i\ ;\quad \bar{\lambda}_{ri}\frac{\partial \bar{T}_i}{\partial \rho}=\bar{\lambda}_{r,i+1}\frac{\partial \bar{T}_{i+1}}{\partial \rho}\quad ;\ i=1\sim(N-1) \tag{5}$$

$$\rho=1\ ;\quad \frac{\partial \bar{T}_1}{\partial \rho}+H_b\bar{T}_N=H_b\bar{T}_b f_b(\theta) \tag{6}$$

$$\theta=0,\ \theta_0\ ;\quad \bar{T}_i=0\quad ;\ i=1\sim N \tag{7}$$

where

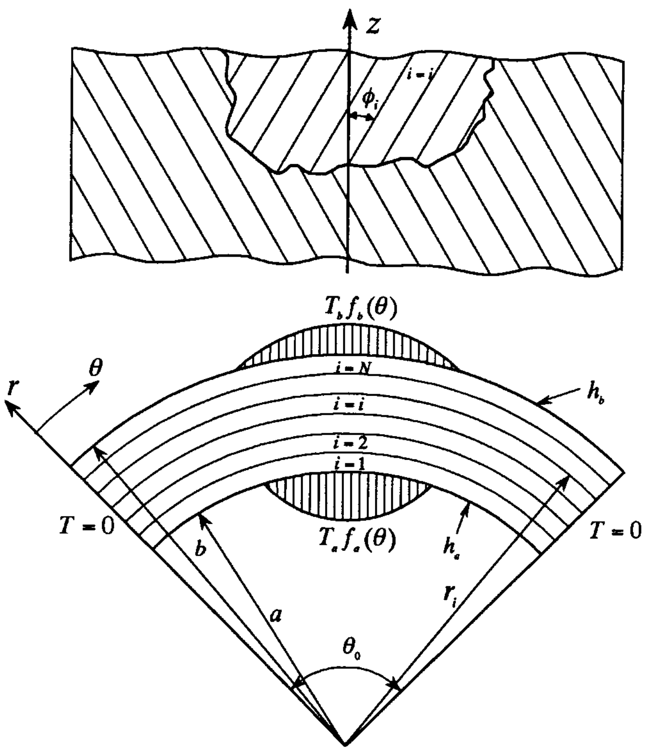

Figure 1 Analytical model and coordinate system

$$\bar{\kappa}_{ri}=\bar{\kappa}_{Ti},\ \bar{\kappa}_{\theta i}=\bar{\kappa}_{Li}\sin^2\phi_i+\bar{\kappa}_{Ti}\cos^2\phi_i,\ \bar{\lambda}_{ri}=\bar{\lambda}_{Ti} \tag{8}$$

In expressions (1)-(8), we have introduced the following dimensionless values :

$$(\bar{T}_i,\ \bar{T}_a,\ \bar{T}_b)=\frac{(T_i,\ T_a,\ T_b)}{T_0},\ (\rho,\ R_i,\ \bar{a})=\frac{(r,\ r_i,\ a)}{b},$$

$$(\bar{\kappa}_{ri},\ \bar{\kappa}_{\theta i},\ \bar{\kappa}_{Li},\ \bar{\kappa}_{Ti})=\frac{(\kappa_{ri},\ \kappa_{\theta i},\ \kappa_{Li},\ \kappa_{Ti})}{\kappa_0},$$

$$(\bar{\lambda}_{ri},\ \bar{\lambda}_{Ti})=\frac{(\lambda_{ri},\ \lambda_{Ti})}{\lambda_0},\ \ \tau=\frac{\kappa_0 t}{b^2},\ (H_a,\ H_b)=(h_a,\ h_b)b \tag{9}$$

where T_i is the temperature change of the ith layer ; κ_{ri} and $\kappa_{\theta i}$ are thermal diffusivities in the r and θ directions, respectively ; λ_{ri} is thermal conductivity in the r direction ; t is time ; and T_0, κ_0, and λ_0 are typical values of temperature, thermal diffusivity and thermal conductivity, respectively. In Eq. (8), the subscripts L and T denote the fiber and transverse directions, respectively. Moreover, $r_i(=1, 2, \cdots, N-1)$ are the coordinates of interface of the laminated cylindrical panel.

Introducing the finite sine transformation with respect to the variable θ and Laplace transformation with respect to the variable τ, the solution of equation (1) can be obtained so as to satisfy the conditions (2)-(7). This solution is shown as follows :

$$\bar{T}_i=\sum_{k=1}^{\infty}\bar{T}_{ik}(\rho,\tau)\sin q_k\theta\ ;\quad i=1\sim N \tag{10}$$

where

$$\bar{T}_{ik}(\rho,\tau)=\frac{2}{\theta_0}\left[\frac{1}{F}(\bar{A}_i'\rho^{\gamma_i}+\bar{B}_i'\rho^{-\gamma_i}) + \sum_{j=1}^{\infty}\frac{2\exp(-\mu_j^2\tau)}{\mu_j\Delta'(\mu_j)}\left\{\bar{A}_iJ_{\gamma_i}(\beta_i\mu_j\rho)+\bar{B}_iY_{\gamma_i}(\beta_i\mu_j\rho)\right\}\right] \tag{11}$$

where $J_\gamma(\ )$ and $Y_\gamma(\ )$ are the Bessel function of the first and second kind of order γ, respectively ; Δ and F are the determinants of $2N\times 2N$ matrix $[a_{kl}]$ and $[e_{kl}]$, respectively ; the coefficients $\bar{A}_i$ and $\bar{B}_i$ are defined as the determinant of the matrix similar to the coefficient matrix $[a_{kl}]$, in which the $(2i-1)$th column or $2i$th column is replaced by the constant vector $\{c_k\}$, respectively ; similarly, the coefficients $\bar{A}_i'$ and $\bar{B}_i'$ are defined as the determinant of the matrix similar to the coefficient matrix $[e_{kl}]$, in which the $(2i-1)$th column or $2i$th column is replaced by the constant vector $\{c_k\}$, respectively. Furthermore, the nonzero elements a_{kl} and c_k of the coefficient matrix $[a_{kl}]$ and the constant vector $\{c_k\}$ are given as follows :

$$a_{11}=\beta_1\mu J_{\gamma_1+1}(\beta_1\mu\bar{a})\ +\ (H_a-\frac{\gamma_1}{\bar{a}})\,J_{\gamma_1}(\beta_1\mu\bar{a}),$$

$$a_{12}=\beta_1\mu Y_{\gamma_1+1}(\beta_1\mu\bar{a})\ +\ (H_a-\frac{\gamma_1}{\bar{a}})\,Y_{\gamma_1}(\beta_1\mu\bar{a}),$$

$$a_{2N,2N-1}=(H_b+\gamma_N)\,J_{\gamma_N}(\beta_N\mu)-\beta_N\mu J_{\gamma_N+1}(\beta_N\mu),$$

$$a_{2N,2N}=(H_b+\gamma_N)\,Y_{\gamma_N}(\beta_N\mu)-\beta_N\mu Y_{\gamma_N+1}(\beta_N\mu),$$

$$a_{2i,2i-1}=J_{\gamma_i}(\beta_i\mu R_i),\ a_{2i,2i}=Y_{\gamma_i}(\beta_i\mu R_i),$$

$$a_{2i,2i+1}=-J_{\gamma_i+1}(\beta_{i+1}\mu R_i),\ a_{2i,2i+2}=-Y_{\gamma_i+1}(\beta_{i+1}\mu R_i),$$

$$a_{2i+1,2i-1}=\bar{\lambda}_{ri}\left\{\frac{\gamma_i}{R_i}J_{\gamma_i}(\beta_i\mu R_i)-\beta_i\mu J_{\gamma_i+1}(\beta_i\mu R_i)\right\},$$

$$a_{2i+1,\,2i}=\bar{\lambda}_{ri}\left\{\frac{\gamma_i}{R_i}Y_{\gamma_i}(\beta_i\mu R_i)-\beta_i\mu Y_{\gamma_i+1}(\beta_i\mu R_i)\right\},$$

$$a_{2i+1,\,2i+1}=-\bar{\lambda}_{r,\,i+1}\left\{\frac{\gamma_{i+1}}{R_i}J_{\gamma_{i+1}}(\beta_{i+1}\mu R_i)-\beta_{i+1}\mu J_{\gamma_{i+1}+1}(\beta_{i+1}\mu R_i)\right\},$$

$$a_{2i+1,\,2i+2}=-\bar{\lambda}_{r,\,i+1}\left\{\frac{\gamma_{i+1}}{R_i}Y_{\gamma_{i+1}}(\beta_{i+1}\mu R_i)-\beta_{i+1}\mu Y_{\gamma_{i+1}+1}(\beta_{i+1}\mu R_i)\right\}$$

$$;\ i=1\sim(N-1) \tag{12}$$

$$c_1=H_a\bar{T}_a\hat{f}_a(q),\ c_{2N}=H_b\bar{T}_b\hat{f}_b(q) \tag{13}$$

On the other hand, the element e_{kl} of the coefficient matrix $[e_{kl}]$ is omitted here for the sake of brevity. In Eq. (13), q represents the parameter of finite sine transformation with respect to the variable θ and a symbol $(\hat{\ })$ represents the image function. In Eqs. (10) and (11), $\Delta'(\mu_j)$, q_k, β_i and γ_i are

$$\Delta'(\mu_j)=\left.\frac{d\Delta}{d\mu}\right|_{\mu=\mu_j},\quad q_k=\frac{k\pi}{\theta_0},\quad \beta_i=\frac{1}{\sqrt{\bar{\kappa}_{ri}}},\quad \gamma_i=\sqrt{\frac{\bar{\kappa}_{\theta i}}{\bar{\kappa}_{ri}}}\,q_k \tag{14}$$

and μ_j represent the jth positive roots of the following transcendental equation

$$\Delta(\mu)=0 \tag{15}$$

2.2 Thermal stress analysis

We now analyse the transient thermal stress of an angle-ply laminated cylindrical panel with simply supported edges as a generalized plane deformation problem. In each layer of the laminated cylindrical panel, the fiber direction, the in-plane transverse direction and the radial direction are denoted by L, T and R, respectively. Each layer has orthotropic material properties between the fiber-reinforced direction and its orthogonal direction. Applying the coordinate transformation rule to stress-strain relations for the ith layer, stress-strain relations for the global coordinate system (r,θ,z) are :

$$\begin{Bmatrix}\bar{\sigma}_{zzi}\\ \bar{\sigma}_{\theta\theta i}\\ \bar{\sigma}_{rri}\\ \bar{\sigma}_{r\theta i}\\ \bar{\sigma}_{rzi}\\ \bar{\sigma}_{\theta zi}\end{Bmatrix}=\begin{bmatrix}\bar{Q}^*_{11i} & \bar{Q}^*_{12i} & \bar{Q}^*_{13i} & 0 & 0 & \bar{Q}^*_{16i}\\ \bar{Q}^*_{12i} & \bar{Q}^*_{22i} & \bar{Q}^*_{23i} & 0 & 0 & \bar{Q}^*_{26i}\\ \bar{Q}^*_{13i} & \bar{Q}^*_{23i} & \bar{Q}^*_{33i} & 0 & 0 & \bar{Q}^*_{36i}\\ 0 & 0 & 0 & \bar{Q}^*_{44i} & \bar{Q}^*_{45i} & 0\\ 0 & 0 & 0 & \bar{Q}^*_{45i} & \bar{Q}^*_{55i} & 0\\ \bar{Q}^*_{16i} & \bar{Q}^*_{26i} & \bar{Q}^*_{36i} & 0 & 0 & \bar{Q}^*_{66i}\end{bmatrix}\begin{Bmatrix}\bar{\varepsilon}_{zzi}-\bar{\alpha}_{zi}\bar{T}_i\\ \bar{\varepsilon}_{\theta\theta i}-\bar{\alpha}_{\theta i}\bar{T}\\ \bar{\varepsilon}_{rri}-\bar{\alpha}_{ri}\bar{T}\\ \bar{\gamma}_{r\theta i}\\ \bar{\gamma}_{rzi}\\ \bar{\gamma}_{\theta zi}-\bar{\alpha}_{\theta zi}\bar{T}_i\end{Bmatrix} \tag{16}$$

where

$$\begin{aligned}
\bar{Q}^*_{11i}&=m_i^4\bar{Q}_{11i}+2m_i^2n_i^2(\bar{Q}_{12i}+\bar{Q}_{66i})+n_i^4\bar{Q}_{22i},\\
\bar{Q}^*_{12i}&=m_i^4\bar{Q}_{12i}+m_i^2n_i^2(\bar{Q}_{11i}+\bar{Q}_{22i}-2\bar{Q}_{66i})+n_i^4\bar{Q}_{12i},\\
\bar{Q}^*_{13i}&=m_i^2\bar{Q}_{12i}+n_i^2\bar{Q}_{23i},\\
\bar{Q}^*_{16i}&=m_in_i\left\{m_i^2(\bar{Q}_{11i}-\bar{Q}_{12i}-\bar{Q}_{66i})+n_i^2(\bar{Q}_{12i}-\bar{Q}_{22i}+\bar{Q}_{66i})\right\},\\
\bar{Q}^*_{22i}&=m_i^4\bar{Q}_{22i}+2m_i^2n_i^2(\bar{Q}_{12i}+\bar{Q}_{66i})+n_i^4\bar{Q}_{11i},\\
\bar{Q}^*_{23i}&=n_i^2\bar{Q}_{12i}+m_i^2\bar{Q}_{23i},\\
\bar{Q}^*_{26i}&=m_in_i\left\{m_i^2(\bar{Q}_{12i}-\bar{Q}_{22i}+\bar{Q}_{66i})+n_i^2(\bar{Q}_{11i}-\bar{Q}_{12i}-\bar{Q}_{66i})\right\},\\
\bar{Q}^*_{36i}&=m_in_i(\bar{Q}_{12i}-\bar{Q}_{23i}),\\
\bar{Q}^*_{44i}&=(m_i^2\bar{Q}_{44i}+n_i^2\bar{Q}_{55i})/2,\\
\bar{Q}^*_{45i}&=m_in_i(\bar{Q}_{55i}-\bar{Q}_{44i})/2,\\
\bar{Q}^*_{55i}&=(n_i^2\bar{Q}_{44i}+m_i^2\bar{Q}_{55i})/2,\\
\bar{Q}^*_{66i}&=m_i^2n_i^2(\bar{Q}_{11i}+\bar{Q}_{22i}-2\bar{Q}_{12i}-\bar{Q}_{66i})+(m_i^4+n_i^4)\bar{Q}_{66i}/2,\\
\bar{Q}^*_{33i}&=\bar{Q}_{22i}
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\bar{\alpha}_{zi}=m_i^2\bar{\alpha}_{Li}+n_i^2\bar{\alpha}_{Ti},\ \bar{\alpha}_{\theta i}=n_i^2\bar{\alpha}_{Li}+m_i^2\bar{\alpha}_{Ti},\ \bar{\alpha}_{ri}=\bar{\alpha}_{Ti},\\
&\bar{\alpha}_{\theta zi}=2m_in_i(\bar{\alpha}_{Li}-\bar{\alpha}_{Ti}),\ m_i=\cos\phi_i,\ n_i=\sin\phi_i
\end{aligned} \tag{18}$$

In expressions (16)-(18), the following dimensionless values are introduced :

$$\bar{\sigma}_{kli}=\frac{\sigma_{kli}}{\alpha_0E_0T_0},\ (\bar{\varepsilon}_{kli},\bar{\gamma}_{kli})=\frac{(\varepsilon_{kli},\gamma_{kli})}{\alpha_0T_0},\ (\bar{\alpha}_{ki},\bar{\alpha}_{\theta zi})=\frac{(\alpha_{ki},\alpha_{\theta zi})}{\alpha_0},$$

$$(\bar{Q}_{kli},\bar{Q}^*_{kli})=\frac{(Q_{kli},Q^*_{kli})}{E_0} \tag{19}$$

where σ_{kli} are the stress components, ε_{kli} are the strain tensor, γ_{kli} are the engineering shear strain components, α_{ki} and $\alpha_{\theta zi}$ are the coefficients of linear thermal expansion, Q_{kli} are the elastic stiffness constants, Q^*_{kli} are the transformed elastic stiffness constants, and α_0 and E_0 are the typical values of the coefficient of linear thermal expansion and Young's modulus of elasticity, respectively.

Next, we assume the displacement components for the global coordinate system in the following forms :

$$\bar{u}_{ri}=\bar{u}_{ri}(\rho,\theta),\quad \bar{u}_{\theta i}=\bar{u}_{\theta i}(\rho,\theta),\ \bar{u}_{zi}=\bar{u}_{zi}(\rho,\theta) \tag{20}$$

where $\bar{u}_{ri}$, $\bar{u}_{\theta i}$ and $\bar{u}_{zi}$ are the dimensionless quantities of displacements in the r, θ and z directions, respectively, and the dimensionless values introduced in Eq. (20) are given by the following relations :

$$(\bar{u}_{ri},\ \bar{u}_{\theta i},\ \bar{u}_{zi})=\frac{(u_{ri},\ u_{\theta i},\ u_{zi})}{\alpha_0T_0b} \tag{21}$$

Taking into account Eq. (20) and substituting the displacement-strain relations and Eq. (16) into the equilibrium equations, the displacement equations of equilibrium are written as

$$\begin{aligned}
&\bar{Q}^*_{33i}(\bar{u}_{ri,\rho\rho}+\rho^{-1}\bar{u}_{ri,\rho})-\rho^{-2}(\bar{Q}^*_{22i}\bar{u}_{ri}-\bar{Q}^*_{44i}\bar{u}_{ri,\theta\theta})\\
&\quad+(\bar{Q}^*_{23i}+\bar{Q}^*_{44i})\rho^{-1}\bar{u}_{\theta i,\rho\theta}-(\bar{Q}^*_{22i}+\bar{Q}^*_{44i})\rho^{-2}\bar{u}_{\theta i,\theta}\\
&\quad+(\bar{Q}^*_{36i}+\bar{Q}^*_{45i})\rho^{-1}\bar{u}_{zi,\rho\theta}-Q^*_{26i}\rho^{-2}\bar{u}_{zi,\theta}\\
&\quad=\bar{\beta}_{ri}\bar{T}_{i,\rho}+(\bar{\beta}_{ri}-\bar{\beta}_{\theta i})\rho^{-1}\bar{T}_i
\end{aligned} \tag{22}$$

$$\begin{aligned}
&(\bar{Q}^*_{44i}+\bar{Q}^*_{23i})\rho^{-1}\bar{u}_{ri,\rho\theta}+(\bar{Q}^*_{44i}+\bar{Q}^*_{22i})\rho^{-2}\bar{u}_{zi,\theta}\\
&\quad+\bar{Q}^*_{44i}(\rho^{-1}\bar{u}_{\theta i,\rho}-\rho^{-2}\bar{u}_{\theta i}+\bar{u}_{\theta i,\rho\rho})+\bar{Q}^*_{22i}\rho^{-2}\bar{u}_{\theta i,\theta\theta}\\
&\quad+\bar{Q}^*_{45i}(\bar{u}_{zi,\rho\rho}+2\rho^{-1}\bar{u}_{zi,\rho})+\bar{Q}^*_{26i}\rho^{-2}\bar{u}_{zi,\theta\theta}=\rho^{-1}\bar{\beta}_{\theta i}\bar{T}_{i,\theta}
\end{aligned} \tag{23}$$

$$\begin{aligned}
&(\bar{Q}^*_{36i}+\bar{Q}^*_{45i})\rho^{-1}\bar{u}_{ri,\rho\theta}+\bar{Q}^*_{26i}\rho^{-2}\bar{u}_{ri,\theta}+\bar{Q}^*_{45i}\bar{u}_{\theta i,\rho\rho}+\bar{Q}^*_{26i}\rho^{-2}\bar{u}_{\theta i,\theta\theta}\\
&\quad+\bar{Q}^*_{55i}(\bar{u}_{zi,\rho\rho}+\rho^{-1}\bar{u}_{zi,\rho})+\bar{Q}^*_{66i}\rho^{-2}\bar{u}_{zi,\theta\theta}=\rho^{-1}\bar{\beta}_{\theta zi}\bar{T}_{i,\theta}
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
\bar{\beta}_{zi}&=\bar{Q}^*_{11i}\bar{\alpha}_{zi}+\bar{Q}^*_{12i}\bar{\alpha}_{\theta i}+\bar{Q}^*_{13i}\bar{\alpha}_{ri}+\bar{Q}^*_{16i}\bar{\alpha}_{\theta zi},\\
\bar{\beta}_{\theta i}&=\bar{Q}^*_{12i}\bar{\alpha}_{zi}+\bar{Q}^*_{22i}\bar{\alpha}_{\theta i}+\bar{Q}^*_{23i}\bar{\alpha}_{ri}+\bar{Q}^*_{26i}\bar{\alpha}_{\theta zi},\\
\bar{\beta}_{ri}&=\bar{Q}^*_{13i}\bar{\alpha}_{zi}+\bar{Q}^*_{23i}\bar{\alpha}_{\theta i}+\bar{Q}^*_{33i}\bar{\alpha}_{ri}+\bar{Q}^*_{36i}\bar{\alpha}_{\theta zi},\\
\bar{\beta}_{\theta zi}&=\bar{Q}^*_{16i}\bar{\alpha}_{zi}+\bar{Q}^*_{26i}\bar{\alpha}_{\theta i}+\bar{Q}^*_{36i}\bar{\alpha}_{ri}+\bar{Q}^*_{66i}\bar{\alpha}_{\theta zi}
\end{aligned} \tag{25}$$

In Eqs. (22)-(24), a comma denotes partial differentiation with respect to the variable that follows. The boundary conditions of inner and outer surfaces and the conditions of continuity on the interfaces can be represented as follows :

$$\begin{aligned}
&\rho=a;\quad \bar{\sigma}_{rr1}=0,\quad \bar{\sigma}_{r\theta 1}=0,\quad \bar{\sigma}_{rz1}=0,\\
&\rho=1;\quad \bar{\sigma}_{rrN}=0,\quad \bar{\sigma}_{r\theta N}=0,\quad \bar{\sigma}_{rzN}=0,\\
&\rho=R_i;\quad \bar{\sigma}_{rri}=\bar{\sigma}_{rr,\,i+1},\quad \bar{\sigma}_{r\theta i}=\bar{\sigma}_{r\theta,\,i+1},\quad \bar{\sigma}_{rzi}=\bar{\sigma}_{rz,\,i+1},\\
&\qquad\quad \bar{u}_{ri}=\bar{u}_{r,\,i+1},\ \bar{u}_{\theta i}=\bar{u}_{\theta,\,i+1},\ \bar{u}_{zi}=\bar{u}_{z,\,i+1}
\end{aligned} \tag{26}$$

The most general edge condition that the both edges are supported is a simply supported condition. We now consider the case of a simply supported panel given by the following relations :

$$\theta=0,\ \theta_0;\quad \bar{\sigma}_{\theta\theta i}=0,\quad \bar{\sigma}_{\theta zi}=0,\quad \bar{u}_{ri}=0 \tag{27}$$

We assume the solutions of Eqs. (22)-(24) in order to satisfy Eq. (27) in the following form.

$$\bar{u}_{ri}=\sum_{k=1}^{\infty}\left\{U_{rcik}(\rho)+U_{rpik}(\rho)\right\}\sin q_k\theta,$$

$$\bar{u}_{\theta i}=\sum_{k=1}^{\infty}\left\{U_{\theta cik}(\rho)+U_{\theta pik}(\rho)\right\}\cos q_k\theta\,,$$

$$\bar{u}_{zi}=\sum_{k=1}^{\infty}\left\{U_{zcik}(\rho)+U_{zpik}(\rho)\right\}\cos q_k\theta \tag{28}$$

In expressions (28), the first term on the right side gives the homogeneous solution and the second term of right side gives the particular solution. We now consider the homogeneous solution and introduce the following equations.

$$\rho=\exp(s) \tag{29}$$

$$(U_{rcik},\,U_{\theta cik},\,U_{zcik})=(U^0_{rcik},\,U^0_{\theta cik},\,U^0_{zcik})\exp(\lambda_i s) \tag{30}$$

Substituting the first term on the right side of Eq. (28) into the homogeneous expression of Eqs. (22)-(24), and later changing a variable with the use of Eq. (29), the condition that non-trivial solutions of $(U^0_{rcik},\,U^0_{\theta cik},\,U^0_{zcik})$ exist leads to the following equation.

$$p_i^3+d_ip_i+f_i=0 \tag{31}$$

where

$$p_i=\lambda_i^2-\frac{B^{(i)}}{3A^{(i)}},\quad d_i=\left[\frac{3A^{(i)}C^{(i)}+(B^{(i)})^2}{3(A^{(i)})^2}\right],$$

$$f_i=-\left[\frac{2(B^{(i)})^3+9A^{(i)}B^{(i)}C^{(i)}+27D^{(i)}(A^{(i)})^2}{27(A^{(i)})^3}\right],$$

$$\begin{aligned}
A^{(i)}&=\bar{Q}^*_{33i}[\bar{Q}^*_{55i}\bar{Q}^*_{44i}-(\bar{Q}^*_{45i})^2],\\
B^{(i)}&=q_k^2\bar{Q}^*_{55i}[\bar{Q}^*_{33i}\bar{Q}^*_{22i}-(\bar{Q}^*_{23i})^2]+q_k^2\bar{Q}^*_{44i}[\bar{Q}^*_{33i}\bar{Q}^*_{66i}-(\bar{Q}^*_{36i})^2]\\
&\quad+(\bar{Q}^*_{33i}+\bar{Q}^*_{22i}-2q_k^2\bar{Q}^*_{23i})[\bar{Q}^*_{55i}\bar{Q}^*_{44i}-(\bar{Q}^*_{45i})^2]\\
&\quad-2q_k^2\bar{Q}^*_{45i}(\bar{Q}^*_{33i}\bar{Q}^*_{26i}-\bar{Q}^*_{23i}\bar{Q}^*_{36i}),\\
C^{(i)}&=[q_k^4\bar{Q}^*_{36i}+2q_k^2(q_k^2-1)\bar{Q}^*_{45i}](\bar{Q}^*_{22i}\bar{Q}^*_{36i}-\bar{Q}^*_{23i}\bar{Q}^*_{26i})\\
&\quad-q_k^4\bar{Q}^*_{33i}[\bar{Q}^*_{22i}\bar{Q}^*_{66i}-(\bar{Q}^*_{26i})^2]+q_k^4(\bar{Q}^*_{23i}+2\bar{Q}^*_{44i})(\bar{Q}^*_{23i}\bar{Q}^*_{66i}-\bar{Q}^*_{36i}\bar{Q}^*_{26i})\\
&\quad-\bar{Q}^*_{22i}(q_k^2-1)^2[\bar{Q}^*_{55i}\bar{Q}^*_{44i}-(\bar{Q}^*_{45i})^2]\\
&\quad+q_k^2\bar{Q}^*_{44i}[(\bar{Q}^*_{36i})^2+(\bar{Q}^*_{26i})^2-\bar{Q}^*_{33i}\bar{Q}^*_{66i}-\bar{Q}^*_{22i}\bar{Q}^*_{66i}],\\
D^{(i)}&=q_k^2(q_k^2-1)^2\bar{Q}^*_{44i}[\bar{Q}^*_{22i}\bar{Q}^*_{66i}-(\bar{Q}^*_{26i})^2]
\end{aligned} \tag{32}$$

We now introduce the following expression.

$$H_i=\frac{f_i^2}{4}+\frac{d_i^3}{27} \tag{33}$$

From Eq. (31), there might be three distinct real roots, three real roots with at least two of them being equal or a real root in conjunction with one pair of conjugate complex roots depending H_i being negative, zero or positive, respectively. For instance, $U_{rcik}(\rho)$, $U_{\theta cik}(\rho)$ and $U_{zcik}(\rho)$ can be expressed as follows when $H_i<0$:

$$U_{rcik}(\rho)=\sum_{J=1}^{3}U^J_{rcik}(\rho),\quad U_{\theta cik}(\rho)=\sum_{J=1}^{3}U^J_{\theta cik}(\rho),$$

$$U_{zcik}(\rho)=\sum_{J=1}^{3}U^J_{zcik}(\rho) \tag{34}$$

where

$$\begin{aligned}
U^J_{rcik}(\rho)&=F^{(i)}_{rJ}\rho^{m_{Ji}}+S^{(i)}_{rJ}\rho^{-m_{Ji}},\\
U^J_{\theta cik}(\rho)&=L_{kiJ}(m_{Ji})F^{(i)}_{rJ}\rho^{m_{Ji}}+L_{kiJ}(-m_{Ji})S^{(i)}_{rJ}\rho^{-m_{Ji}},\\
U^J_{zcik}(\rho)&=R_{kiJ}(m_{Ji})F^{(i)}_{rJ}\rho^{m_{Ji}}+R_{kiJ}(-m_{Ji})S^{(i)}_{rJ}\rho^{-m_{Ji}},
\end{aligned}$$

$$m_{Ji}=\sqrt{p_{iJ}+\frac{B^{(i)}}{3A^{(i)}}}\quad \text{if } p_{iJ}+\frac{B^{(i)}}{3A^{(i)}}>0 \tag{35}$$

$$\begin{aligned}
U^J_{rcik}(\rho)&=F^{(i)}_{rJ}\cos(m_{Ji}\ln\rho)+S^{(i)}_{rJ}\sin(m_{Ji}\ln\rho),\\
U^J_{\theta cik}(\rho)&=\{\mathrm{Re}[L_{kiJ}(jm_{Ji})]\cos(m_{Ji}\ln\rho)\\
&\quad-\mathrm{Im}[L_{kiJ}(jm_{Ji})]\sin(m_{Ji}\ln\rho)\}F^{(i)}_{rJ}\\
&\quad+\{\mathrm{Im}[L_{kiJ}(jm_{Ji})]\cos(m_{Ji}\ln\rho)\\
&\quad+\mathrm{Re}[L_{kiJ}(jm_{Ji})]\sin(m_{Ji}\ln\rho)\}S^{(i)}_{rJ},\\
U^J_{zcik}(\rho)&=\{\mathrm{Re}[R_{kiJ}(jm_{Ji})]\cos(m_{Ji}\ln\rho)\\
&\quad-\mathrm{Im}[R_{kiJ}(jm_{Ji})]\sin(m_{Ji}\ln\rho)\}F^{(i)}_{rJ}\\
&\quad+\{\mathrm{Im}[R_{kiJ}(jm_{Ji})]\cos(m_{Ji}\ln\rho)\\
&\quad+\mathrm{Re}[R_{kiJ}(jm_{Ji})]\sin(m_{Ji}\ln\rho)\}S^{(i)}_{rJ},
\end{aligned}$$

$$m_{Ji}=\sqrt{-\left(p_{iJ}+\frac{B^{(i)}}{3A^{(i)}}\right)}\quad \text{if } p_{iJ}+\frac{B^{(i)}}{3A^{(i)}}<0 \tag{36}$$

In Eqs. (35) and (36),

$$\begin{aligned}
L_{kiJ}(x)&=\frac{1}{I_{kiJ}(x)}\{-\bar{Q}^*_{33i}\bar{Q}^*_{45i}(x^4+x^3)+[\bar{Q}^*_{33i}\bar{Q}^*_{26i}q_k^2\\
&\quad+\bar{Q}^*_{45i}(\bar{Q}^*_{22i}+q_k^2\bar{Q}^*_{44i})-q_k^2(\bar{Q}^*_{36i}+\bar{Q}^*_{45i})(\bar{Q}^*_{44i}+\bar{Q}^*_{23i})]x^2\\
&\quad+[\bar{Q}^*_{45i}(\bar{Q}^*_{22i}+q_k^2\bar{Q}^*_{44i})+q_k^2\bar{Q}^*_{26i}(\bar{Q}^*_{44i}+\bar{Q}^*_{23i})\\
&\quad-q_k^2(\bar{Q}^*_{22i}+\bar{Q}^*_{44i})(\bar{Q}^*_{36i}+\bar{Q}^*_{45i})]x+\bar{Q}^*_{26i}\bar{Q}^*_{44i}q_k^2(1-q_k^2)\},\\
R_{kiJ}(x)&=\frac{1}{I_{kiJ}(x)}\{\bar{Q}^*_{33i}\bar{Q}^*_{44i}x^4+[q_k^2(\bar{Q}^*_{23i}+\bar{Q}^*_{44i})^2-\bar{Q}^*_{33i}(q_k^2\bar{Q}^*_{22i}+\bar{Q}^*_{44i})\\
&\quad-\bar{Q}^*_{44i}(\bar{Q}^*_{22i}+q_k^2\bar{Q}^*_{44i})]x^2+\bar{Q}^*_{22i}\bar{Q}^*_{44i}(1-q_k^2)^2\},\\
I_{kiJ}(x)&=q_k\{(\bar{Q}^*_{36i}\bar{Q}^*_{44i}-\bar{Q}^*_{23i}\bar{Q}^*_{45i})x^3+(\bar{Q}^*_{22i}\bar{Q}^*_{45i}-\bar{Q}^*_{23i}\bar{Q}^*_{45i}\\
&\quad-\bar{Q}^*_{26i}\bar{Q}^*_{44i})x^2+[q_k^2\bar{Q}^*_{26i}(\bar{Q}^*_{23i}+\bar{Q}^*_{44i})+\bar{Q}^*_{45i}(\bar{Q}^*_{22i}+\bar{Q}^*_{44i})\\
&\quad-(\bar{Q}^*_{36i}+\bar{Q}^*_{45i})(q_k^2\bar{Q}^*_{22i}+\bar{Q}^*_{44i})]x+\bar{Q}^*_{26i}\bar{Q}^*_{44i}(1-q_k^2)\}
\end{aligned} \tag{37}$$

In Eq. (36), j, Re[ ] and Im[ ] are imaginary unit $j=\sqrt{-1}$, real part and imaginary part, respectively. Furthermore, in Eqs. (35) and (36), $F^{(i)}_{rJ}$ and $S^{(i)}_{rJ}$ are unknown constants. The case of $H_i=0$ or $H_i>0$ is omitted here for the sake of brevity.

In order to obtain the particular solution, we use the series expansions of the Bessel functions. Since the order γ_i of the Bessel function in Eq. (11) is not integer in general except $\phi_i=0$, Eq. (11) can be written as the following expression.

$$\bar{T}_{ik}(\rho,\tau)=\sum_{n=0}^{\infty}\left[a_{ni}(\tau)\rho^{2n+\gamma_i}+b_{ni}(\tau)\rho^{2n-\gamma_i}\right] \tag{38}$$

Expressions for the functions $a_{ni}(\tau)$ and $b_{ni}(\tau)$ in Eq. (38) are omitted here for the sake of brevity. $U_{rpik}(\rho)$, $U_{\theta pik}(\rho)$ and $U_{zpik}(\rho)$ of the particular solution are obtained as the function systems like Eq. (38).

Then, the stress components can be evaluated by substituting Eq. (28) into the displacement-strain relations, and later into the stress-strain relations. The unknown constants in Eqs. (35) and (36) are determined so as to satisfy the boundary condition (26).

3. Numerical results

To illustrate the foregoing analysis, we consider the angle-ply laminated cylindrical panel composed of alumina fiber reinforced aluminum composite, with the following properties [11] :

$$\begin{aligned}
&\kappa_L=41.1\times10^{-6}\ m^2/s,\ \kappa_T=29.5\times10^{-6}\ m^2/s,\\
&\alpha_L=7.6\times10^{-6}\ 1/K,\ \alpha_T=14.0\times10^{-6}\ 1/K,\\
&\lambda_L=105\ W/mK,\ \lambda_T=75\ W/mK,\ E_L=150\ GPa,\ E_T=110\ GPa,\\
&G_{LT}=35\ GPa,\ G_{TT}=41\ GPa,\ \nu_{LT}=0.33,\ \nu_{TT}=0.33,\ \nu_{TL}=0.242
\end{aligned} \tag{39}$$

where G and ν are the shear modulus of elasticity and Poisson's ratio, respectively. We assume that each layer of laminated cylindrical panel consists of the same orthotropic material, and consider a 2-layered anti-symmetric angle-ply laminated cylindrical panel with the fiber orientation $(\phi_0,-\phi_0)$ and the same thickness. The thermal and mechanical behaviors when the laminated cylindrical panel is heated from an inner surface are different to those when the laminated cylindrical panel is heated from an outer surface. Here we assume that the laminated cylindrical panel is heated from the inner surface by surrounding media, the temperature of which is denoted by the symmetric function with respect to the center of panel ($\theta=\theta_0/2$). The numerical parameters of heat conduction and shape are presented as follows :

$$H_a=H_b=5.0,\ \bar{T}_a=1,\ \bar{T}_b=0,\ \theta_0=90°,\ \bar{a}=0.5\sim0.8,\ \phi_0=60°,$$
$$f_a(\theta)=(1-\theta'^2/\theta_a^2)H(\theta_a-|\theta'|),\ \theta'=\theta-\theta_0/2,\ \theta_a=15° \quad (40)$$

where $H(\ )$ is Heaviside's function. The typical values of material properties such as κ_0, λ_0, α_0 and E_0, used to normalize the numerical data, are based on those of fiber direction.

The numerical results for radius ratio $\bar{a}=0.7$ are shown in Figures 2-7. Figure 2 shows the results of temperature change. The distribution in a transient state ($\tau=0.01$) is shown in Figure 2(a) and the distribution in a steady state ($\tau=\infty$) shown in Figure 2(b). As shown in Figure 2, the temperature rise can clearly be seen in the heated region. Figure 3 shows the variation of the thermal displacement $\bar{u}_r$ on the heated surface. As shown in Figure 3, the absolute value of thermal displacement $\bar{u}_r$ rises as the time

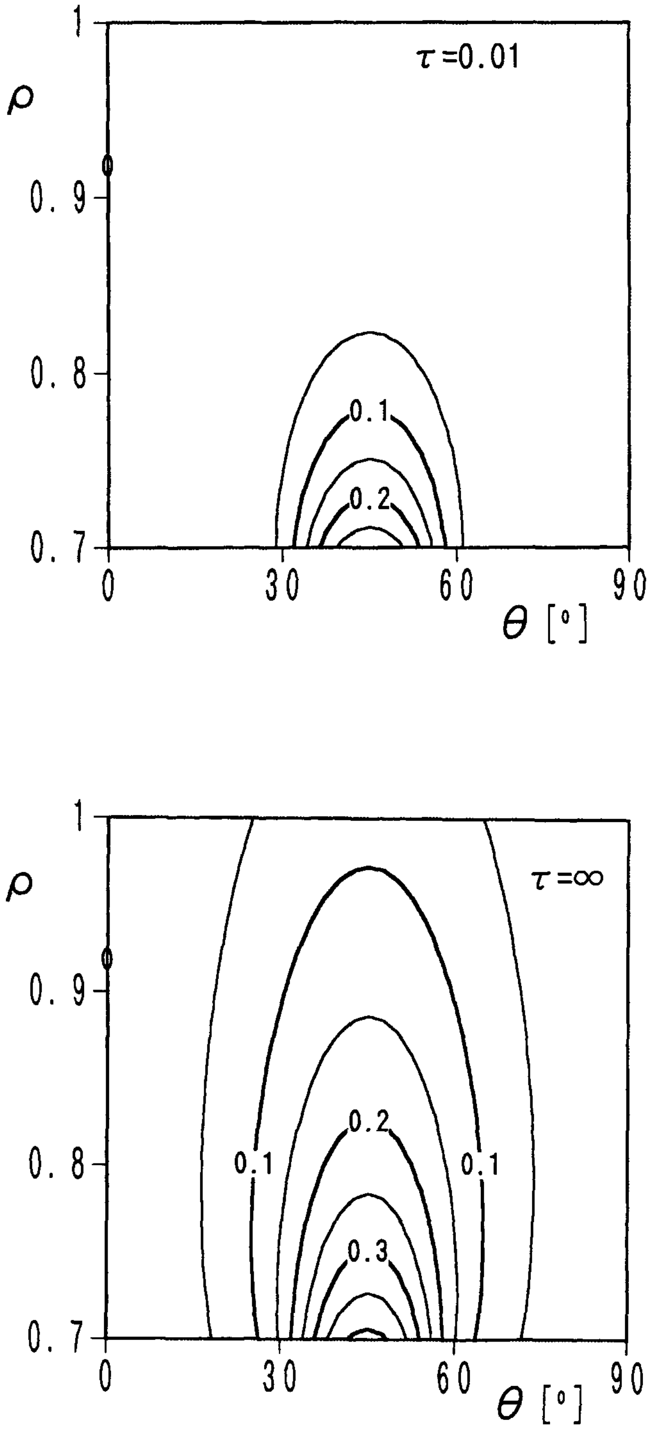

Figure 2 Temperature change ($\bar{a}=0.7$)
(a) distribution in a transient state ($\tau=0.01$)
(b) distribution in a steady state ($\tau=\infty$)

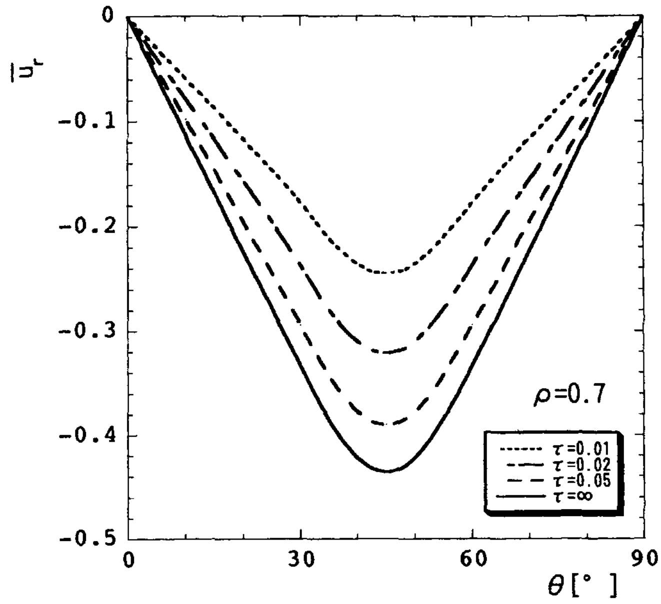

Figure 3 Variation of thermal displacement $\bar{u}_r$ on the heated surface ($\rho=\bar{a}$, $\bar{a}=0.7$)

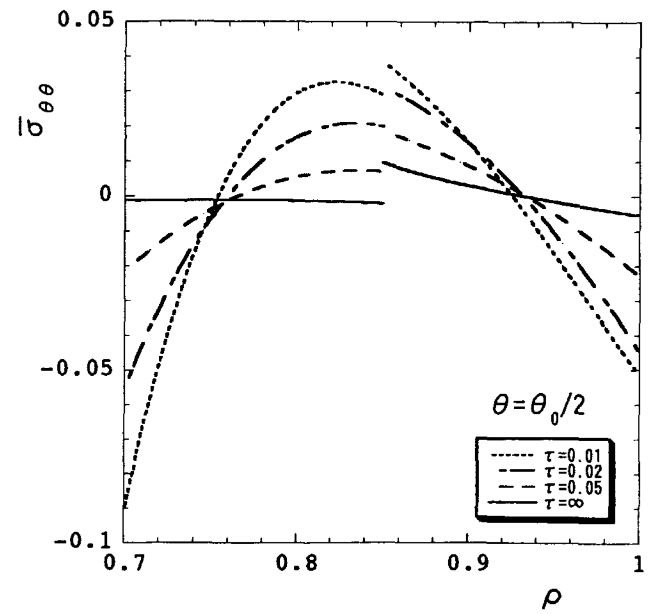

Figure 4 Variation of thermal stress $\bar{\sigma}_{\theta\theta}$ in the radial direction ($\theta=\theta_0/2$, $\bar{a}=0.7$)

proceeds and has a maximum value in the steady state. Figure 4 shows the variation of the normal stress $\bar{\sigma}_{\theta\theta}$ in the radial direction at the midpoint of the panel. From Figure 4, discontinuities of stress occur on the interface ($\rho = 0.85$). In order to valuate the phenomenon of delamination, it is necessary to focus attention on the transverse stress components. Figure 5 shows the variation of the normal stress $\bar{\sigma}_{rr}$. The variation on the interface is shown in Figure 5(a) and the variation in the radial direction at the midpoint of the panel is shown in Figure 5(b). As shown in Figure 5, it can be seen that the stress variation becomes significant with the progress of time and maximum tensile stress occurs at the midpoint of the panel and near $\rho = 0.88$ in side of panel in a transient state. Figure 6 shows the variation of the shearing stress $\bar{\sigma}_{rz}$. The distribution in a steady state is shown in Figure 6(a) and the variation on the interface is shown in Figure 6(b). Since the shearing stress $\bar{\sigma}_{rz}$ is anti-symmetric with respect to $\theta = 45^\circ$ under the condition of Eq. (40), Figure 6(a) shows the range $0-45^\circ$. From Figure 6(a), it can be seen that the shearing stress $\bar{\sigma}_{rz}$ shows the maximum value on the interface. As shown in Figure 6(b), the value of shearing stress $\bar{\sigma}_{rz}$ rises as the time proceeds and has a

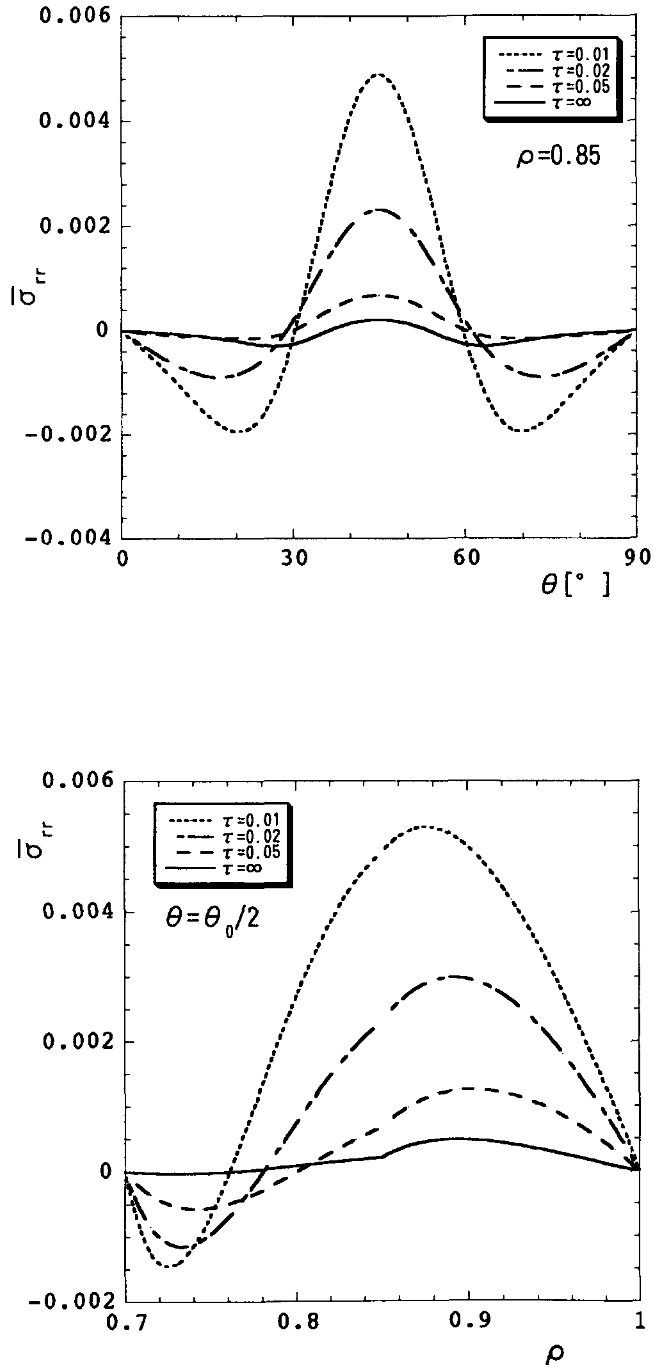

Figure 5 Thermal stress $\bar{\sigma}_{rr}$ ($\bar{a} = 0.7$)
(a) variation on the interface ($\rho = 0.85$)
(b) variation in the radial direction ($\theta = \theta_0/2$)

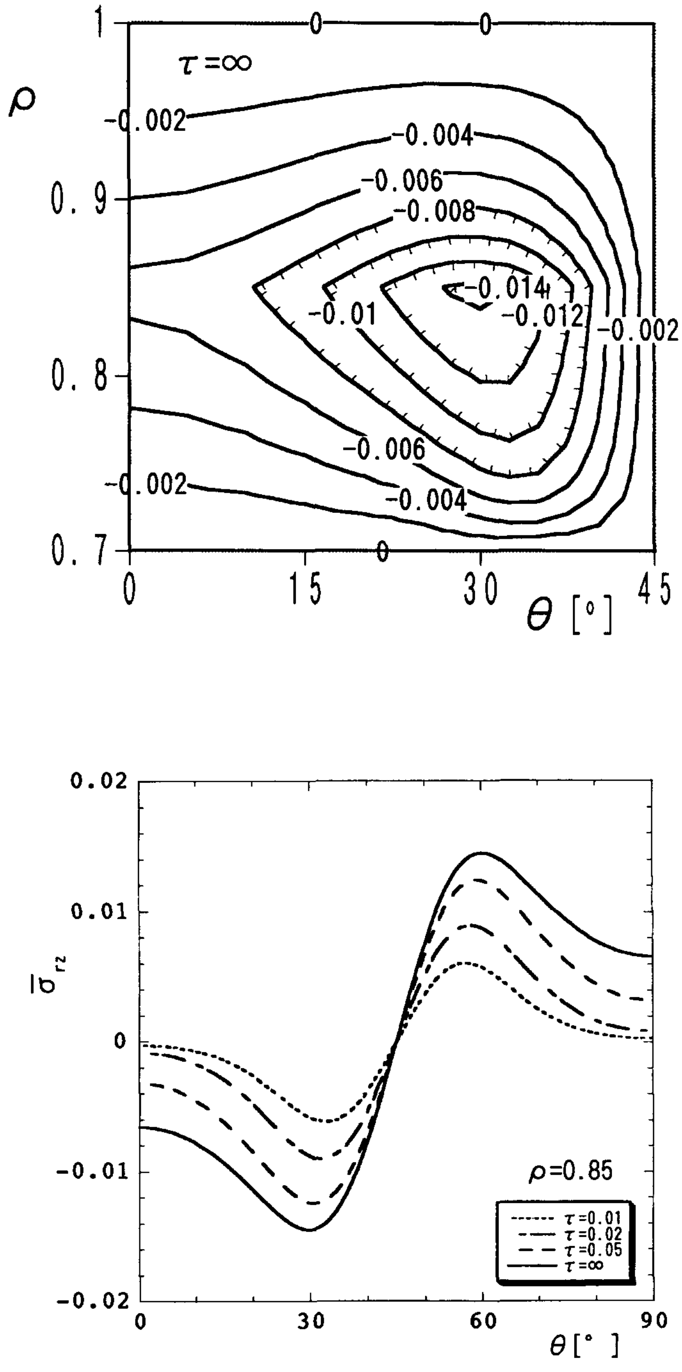

Figure 6 Thermal stress $\bar{\sigma}_{rz}$ ($\bar{a} = 0.7$)
(a) distribution in a steady state ($\tau = \infty$)
(b) variation on the interface ($\rho = 0.85$)

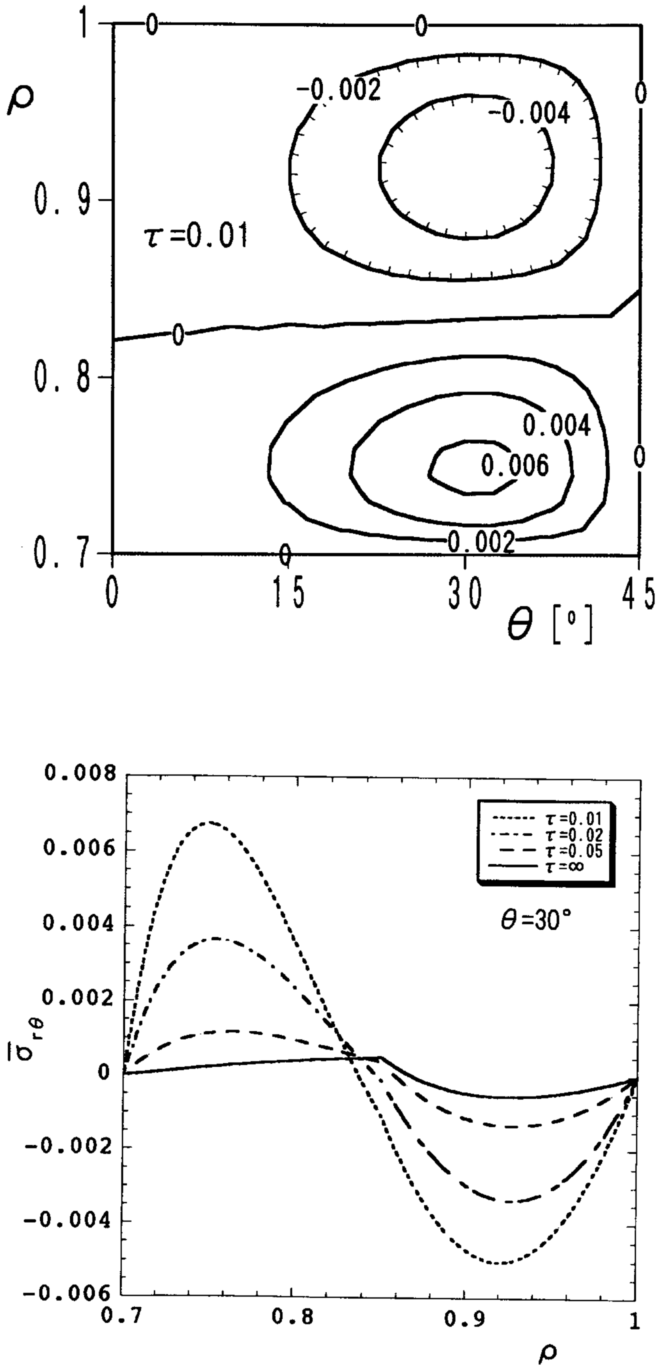

Figure 7 Thermal stress $\bar{\sigma}_{r\theta}(\bar{a}=0.7)$
(a) distribution in a transient state ($\tau=0.01$)
(b) variation in the radial direction ($\theta=30°$)

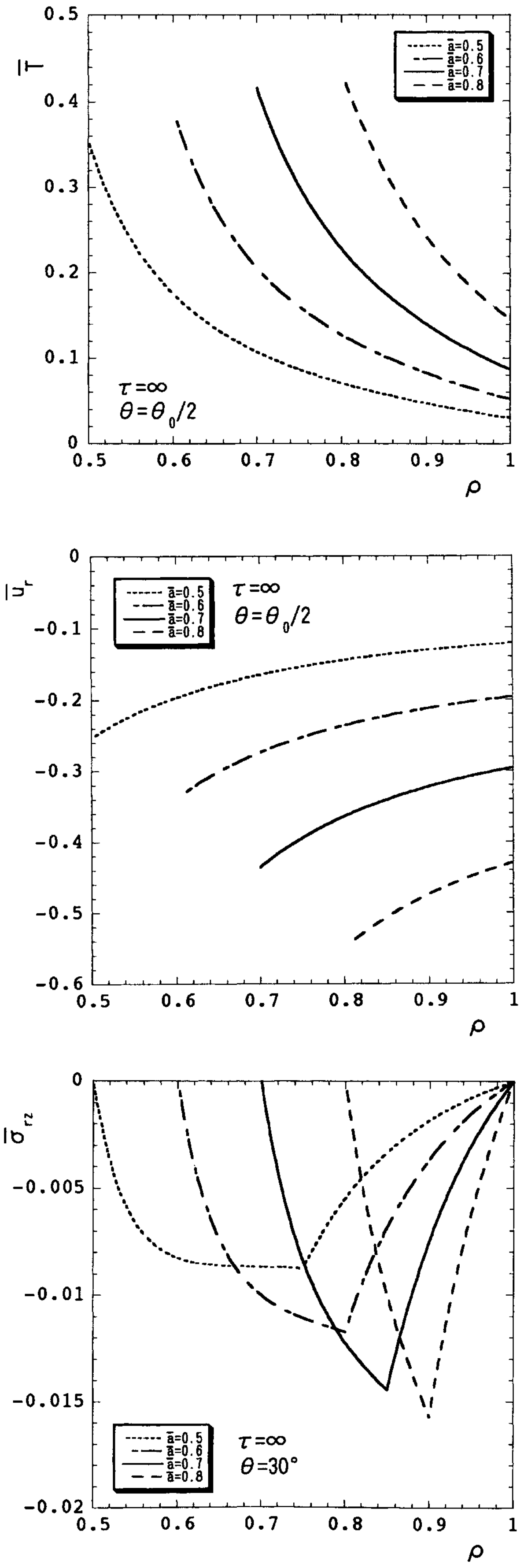

Figure 8 Influence of radius ratio $\bar{a}$
(a) temperature change ($\theta=\theta_0/2$, $\tau=\infty$)
(b) thermal displacement $\bar{u}_r(\theta=\theta_0/2$, $\tau=\infty)$
(c) thermal stress $\bar{\sigma}_{rz}(\theta=30°$, $\tau=\infty)$

maximum value in a steady state. Figure 7 shows the results of the shearing stress $\bar{\sigma}_{r\theta}$. The distribution in a transient state ($\tau = 0.01$) is shown in Figure 7(a) and the variation in the radial direction at the edge ($\theta = 30^\circ$) of heated region is shown in Figure 7(b). Since the shearing stress $\bar{\sigma}_{r\theta}$ is anti-symmetric with respect to $\theta = 45^\circ$ as same as the shearing stress $\bar{\sigma}_{rz}$, Figure 7(a) shows the range 0－45°. As shown in Figure 7(a), it can be seen that the maximum stress occurs near $\theta = 30^\circ$ in side of panel. From Figure 7(b), it can be seen that the shearing stress $\bar{\sigma}_{r\theta}$ shows the maximum value near ρ=0.75 in a transient state.

In order to examine the influence of the radius ratio $\bar{a}$, the variations of temperature change, thermal displacement $\bar{u}_r$ and thermal stress $\bar{\sigma}_{rz}$ in a steady state are shown in Figures. 8(a), 8(b) and 8(c), respectively. Figures. 8(a) and 8(b) show the variations in the radial direction at the midpoint of the panel, and Figure 8(c) shows the variation in the radial direction on the cross section $\theta = 30^\circ$. It can be seen from Figure 8, that the absolute values of $\bar{T}$, $\bar{u}_r$ and $\bar{\sigma}_{rz}$ increase when the radius ratio $\bar{a}$ increases.

4. Conclusions

In the present article, we obtained the exact solution for the transient temperature and transient thermal stresses of an angleply laminated cylindrical panel with simply supported edges due to a nonuniform heat supply in the circumferential direction. Numerical calculations were carried out for a 2-layered antisymmetric angle-ply laminated cylindrical panel composed of alumina fiber reinforced aluminum composite, which is heated from the inner surface. Though numerical calculation were carried out for a 2-layered anti-symmetric angle-ply laminated cylindrical panel which shows a characteristic of fiber orientation angle clearly, numerical calculation for hybrid laminated cylindrical panel with an arbitrary number of layer and arbitrary fiber orientation angles can be carried out. Moreover, transverse shearing stresses and normal stress in the radial direction, which are considered to induce delamination on the interface of each layer, are evaluated precisely in a transient state.

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