Original

Received November 1, 2001
Accepted for Publication March 20, 2002

Computational Inverse Techniques for Material Characterization Using Dynamic Response

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Computational inverse techniques are presented for inverse problems in the areas of material characterization using dynamic response in anisotropic inhomogeneous plates. In these techniques, the inverse problems are formulated into parameter identification problems, in which a set of parameters corresponding to the characteristics of material property can be found by minimizing error functions formulated using the measured displacement responses and that computed using forward solvers based on projected candidates of parameters. The forward solvers used in this work are the hybrid numerical method (HNM) and the finite element method (FEM). The HNM has been proven very effective for transient wave response analysis. The high efficiency of these two forward solvers paves the way for the inverse procedure to solve the inverse problems using waves. Three types of optimization algorithms: nonlinear least square method, evolutional method (genetic algorithm), and their combined method, as well as an identification technique: artificial intelligence method (neural network), are employed as the inverse procedure. Several application examples are presented. It is demonstrated in this work that these inverse problems in material characterization can be solved to a desired accuracy with high efficiency through innovative use of advanced computational techniques.

Key Words : Inverse problem, Material characterization, Dynamic response, Optimization, Wave propagation

1 INTRODUCTION

The accurate knowledge of material property of composites has great importance in many engineering problems of product and system design and quality assurance. The determination of material parameters for composites is much more complicated than for isotropic materials. Conventional experimental methods using direct measurement of strain fields meet with severe difficulties when used to the composite materials. Some specific problems such as boundary effects, sample size dependencies and the presence of non-uniform stress/strain fields often occur in such an experiment and leads to unreliable results. With these difficulties of performing conventional measurement on composites, new effective techniques should be sought. Among those methods proposed, the NDE of composite material properties by employing inverse techniques appears more promising.

Advanced non-destructive methods for material characterization of composite utilize the complex relationship between the structure behaviors and the material property. This relationship is often represented by a known mathematical model, which defines the forward problem. Thus, if a set of reasonably accurate experimentally measured structure behavior data is available, material property of the composite may be identified by solving an inverse problem properly formulated. The material property can be characterized by minimizing the sum of the squares of the deviations between the experimental and the calculated structure behavior data. Ultrasonic wave velocity is often used as the structure behavior data for determination of elastic constants of many anisotropic composites. Rokhin and co-workers [1-3] proposed several modifications of the immersion ultrasonic technique to determinate the elastic constants of composites. Other ultrasonic techniques such as the guided-wave technique [4], the surface-wave technique [5] and the resonance measurements technique [6] are proposed for the determination of elastic constants. The point-source/point-receiver (PS/PR) technique is a procedure to measure the group velocity. The principle of the PS/PR technique is that the group velocities of waves that travel in the direction between the point source (PS) and the point receiver (PR) in a specimen can be determined from the measured wave arrival times. Sachse and Kim [7] described the principle of the PS/PR technique in detail. Balasubramaniam and Rao [8] investigated the reconstruction of material stiffness properties of unidirectional fiber-reinforced composites from obliquely incident ultrasonic bulk wave data. Genetic algorithms (GAs) were used as the inversion technique and detail discussion on advantage as well as disadvantage of GAs for the identification problem over conventional methods were presented. Rok Sribar [9] estimated the elastic constants of composite from the group velocity data using artificial neural network.

In this paper, computational inverse techniques are presented to characterize the material property of composite plates from the dynamic displacement response on the surface of the plates. In this work, the input data used for inverse procedure are the time history of displacements on the surface of an FGM plate, which can be easily, measured using conventional experimental techniques. Numerical models of finite element method (FEM) and hybrid numerical method (HNM) [10, 11] are adopted for forward analysis of structural responses for given candidate of material parameters. The real parameters are such chosen that makes the numerical results match well with the measurements. An objective function is chosen to be the error function formulated using the sum of the squares of the deviations between the experimental measurements and numerical results. Computational optimization techniques are applied to minimize the error function with respect to the material parameters. Many optimization techniques can be used to minimize the error function. These techniques can be classified largely into two groups, namely gradient-based algorithms and evolutional algorithms. The gradient-based algorithms are usual accurate and efficient, but have a high probability to be stuck at a local optimum. The evolutional algorithms like GAs are random searching in nature, and capable of finding the global optimum. Another advantage is the ability to treat discontinuous objective functions. However, evolutional algorithms are generally more computationally expensive. Neural network (NN) is also used for the inverse identification of material property identification.

This paper reports some of the inverse techniques using wave dynamic responses to identify the material parameters of composite materials. Nonlinear least squares method (LSM), genetic algorithms (GAs) and their combination are employed [12-17] in the inverse procedures. An NN model [18, 19] is also trained to characterize the material property of composites. Several application examples are presented to demonstrate the efficiency of these procedures.

2 Statement of the problem

It is aimed to inversely determinate the material properties of composite plates from the measured displacement response on the surface of the plates. In this paper, the measured response is simulated by the computer-generated displacement response based on actual material property of composite plate. The incident excitation wave to the composite plate is assumed to be a vertical line load acting at x=0 on the upper surface, as shown in Figure 1. The line load is independent of the y axis, but as a function of t as

$$f(t)=\begin{cases}\sin(2\pi t/t_d) & 1<t<t_d \\ 0 & t\leq 0 \text{ and } t\geq t_d\end{cases} \qquad (1)$$

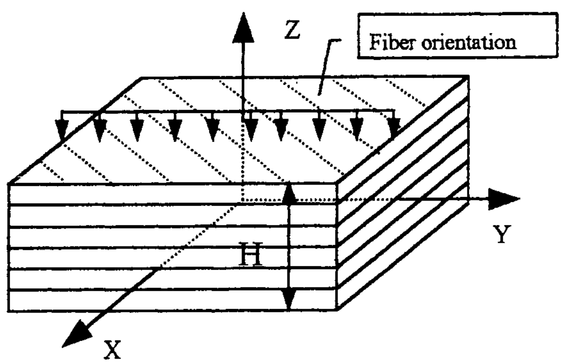

Figure 1 : A laminated plate subjected to a line load on the surface

where t_d is the time duration of the incident wave. Eq. (1) implies that the time history of the incident wave is one cycle of the sine function.

The receiving point is arbitrarily chosen on the surface of composite plate and the displacement response subjected to the line load excitation can be obtained using the forward calculation technique, the hybrid numerical method (HNM).

Using the forward problem solver, the displacement response of the composite plate can be obtained using assumed material properties. The obtained response is, in general, different from those calculated for an actual composite plate. The inverse procedure can then be formulated by an optimization procedure, which minimizes the sum of squares of deviations between calculated for trial material property and for the actual material property. The optimization problem can be stated as follows:

Minimize the objective function of error defined by

$$err(p)=\sum_{i=1}^{M}\left\|u_i^m-u_i^c(p)\right\|^2 \qquad (2)$$

where, p represents the trial material property, u_i^m is the displacement response obtained from HNM for actual material property. The displacement response data set can be created conveniently, using the displacements either at different positions or at different time sequences. M is the number of times (or locations) when (or where) the displacement response is sampled. In this study, the time history of displacement response at one received point is used.

The material property can be determined by solving the optimal problem to minimize the objective function defined in Eq. (2). In an adaptive way, the material property can be determined by solving the identification problem as defined by Eq. (2) via the NN model. The forward analysis techniques and the inverse computation techniques will be illustrated in the following sections.

3 Forward analysis

3.1 HNM for Wave Response Analysis of Composite Plates

In a forward calculation, one needs to calculate the wave fields in a composite plate subjected to an incident wave for given material properties. Both efficiency and accuracy are very important, as many times of forward computations may have to be carried out in the later inverse process. The hybrid numerical method [10, 11] is used in this work as the forward calculator, due to its outstanding efficiency. A brief description of the formulation of the HNM is given as follows.

Consider a composite plate with any number of anisotropic layers in the thickness direction. The thickness of the plate is denoted by H. The plate is divided into N layered elements. A set of approximate partial differential equations for an element is obtained by using the principle of virtual work. Assembling the matrices of adjacent elements, we obtain the dynamic equilibrium equation of the whole plate

$$\mathsf{F}(x,y,t)=\mathsf{K}_D\mathsf{d}(x,y,t)+\mathsf{M}\ddot{\mathsf{d}}(x,y,t) \tag{3}$$

where F is the external force vector acting on the nodal planes that divide the plate into layered elements, and d is the displacement vector on the nodal planes. The matrix K_D is a differential operator matrix for the plate.

Introducing the Fourier transformations with respect to the horizontal coordinates x and y as follows:

$$\tilde{\mathsf{d}}(k_x,k_y,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathsf{d}(x,y,t)e^{ik_x x}e^{ik_y y}dxdy \tag{4}$$

where the real transformation parameters k_x are k_y the wave numbers corresponding to the horizontal coordinates x and y respectively.

The application of Eq. (4) to Eq. (3), leads to a set of dynamic equilibrium equations for the whole plate:

$$\tilde{\mathsf{F}}=\mathsf{M}\ddot{\tilde{\mathsf{d}}}+\mathsf{K}\tilde{\mathsf{d}} \tag{5}$$

where $\tilde{\mathsf{F}}$, $\ddot{\tilde{\mathsf{d}}}$ and $\tilde{\mathsf{d}}$ are the Fourier transformations of, F, $\ddot{\mathsf{d}}$ and d, respectively.

Using the model analysis, the displacement vector $\tilde{\mathsf{d}}$ in the Fourier transformation domain can be obtained; finally the displacement response in the space-time domain can be obtained using the inverse Fourier transformation [20, 21].

For FGM plate, the modified HNM is employed to calculation the displacement responses. In this method, the HNM is modified to accommodate a linear variation of material properties in an element in the thickness direction as follows:

$$(c_{ij})_n=((c_{ij}^u)_n-(c_{ij}^l)_n)\frac{z}{h_n}+(c_{ij}^u)_n \tag{6}$$

$$\rho_n=(\rho_n^u-\rho_n^l)\frac{z}{h_n}+\rho_n^l \tag{7}$$

where, h_n is the thickness of the nth element, $c_{ij}^u=(c_{ij})_n^l$, ρ_n^l, $c_n^u=(c_{ij})_n^u$ and ρ_n^u $(i,j=1,\cdots,6)$ are the elastic coefficient matrix and the mass density on the lower and upper surfaces of the nth element, respectively. Then the dynamic equilibrium equation of the whole FGM plates same as Eq. (5) can be obtained [11].

3.2 FEM for Dynamic Responses

Three-dimensional finite-element software package, LS_DYNA is used as an efficient forward solver for transient analysis of composites plates subject to a dynamic load at an arbitrary location on the laminate surface. Using the FEM, boundary support condition of the plate can be easily considered. The laminate is discretized into eight-node thick shell elements with one point reduced integration formulation. In the thickness direction, one through thickness integration point is used for each material layer. Explicit solver that employs central difference integration algorithm is used for the numerical calculation of the equations of motion in LS_DYNA. Due to the use of explicit solver and one point reduced integration formulation, the response calculation of composite laminate subjected to impact loading can be performed very efficiently for complex structures with complex boundary conditions.

4 Inverse Techniques

4.1 Nonlinear Least Squares Method

The error function is given as the sum of nonlinear squares, as presented in Eq. (2). The material characterization problem can be solved as an optimization problem. A nonlinear least squares method (LSM) proposed by Gill [22] is used, which combines Gauss-Newton and modified Newton algorithm.

4.2 Genetic Algorithms

Genetic algorithms (GAs) are methods to search a set of parameters in parameter space that optimizes the objective function based on the Darwinian principle of natural selection. An initial population sampled randomly from the search space is first created. The fitness is evaluated for each of the individuals based on the evaluation criterion that is the error function. Successive new generation is created through selection, crossover and mutation operations process until the best-fit individual is obtained. The μGA [23] is one of the widely used GAs that uses very small population. It evolves in normal GA fashion and converges in a few generations. A new random population is then chosen while keeping the best individual from the previously converged generation and the evolution process restarts. The elitism strategy that keeps the best individual is applied to force the best individual always passed on to the next generation. It is found that a μGA can avoid premature convergence and demonstrate fast onvergence to the near-optimal region. A μGA also reduces significantly the numbers of the fitness function evaluation because of small population. This is very important for very large and complex systems that require finite element model in the forward analysis. Also there is no need for mutation process in a μGA because new random population is produced after a few generations to prevent the search from convergence to a non-optimal solution.

4.3 Combined GA and LSM method

The advantage of LSM is that it converges very fast to an optimum, especially when the initial guess is close to the optimum and the accuracy can be very high. However, the search for suitable initial points for locally converged optimization method often proves to be difficult. On the other hand, GAs hold complementary promises with respect to LSMs. The GA's advantages are the capability to escape local optima and no need for initial guesses. GAs are, however, computationally expensive for a high accuracy results as their performance at the later stage of searching to be very slow compared to LSMs due to random nature. This fact can often be observed from the convergence curve of GAs, which converges very fast at the beginning and very slow at later stage. It is expected that a combination of a GA and an LSM may provide an ideal performance for the optimization procedure needed for non-linear optimization problems in both efficiency and accuracy.

A combined GA and LSM method is proposed. In this method, GA is firstly used to determine the initial points. The main purpose is to select a set of better solutions close to the optima. The selection criterion is imposed to limit the function value below a required value. Second, these sets of better solutions are used as the initial points in applying the nonlinear LSM. All these solutions from nonlinear LSM searching can be considered as the local optima of the function, and the global optimum is found from these solutions simply by comparing their corresponding error function values. This method is outlined in Figure 2.

4.4 An NN Process

An NN model is referred as a type of computational models that consists of hidden layers of neurons connected between the input and output neurons. The connections between the neurons are described by weights which are to be determined through a process of NN training. The nonlinear hyperbolic functions are usually used as the activation function to increase the modeling flexibility. The neural network is trained with a modified back-propagation (BP) training algorithm. This learning algorithm can overcome the possible saturation of the sigmoid function and speed up the training process of the NN model.

Table 1 : Material properties of SiC and C monolith materials

Material constants	E (GPa)	ν	ρ(g/cm^3)
CVD-SiC	320	0.3	3.22
CVD-C	28	0.3	1.78

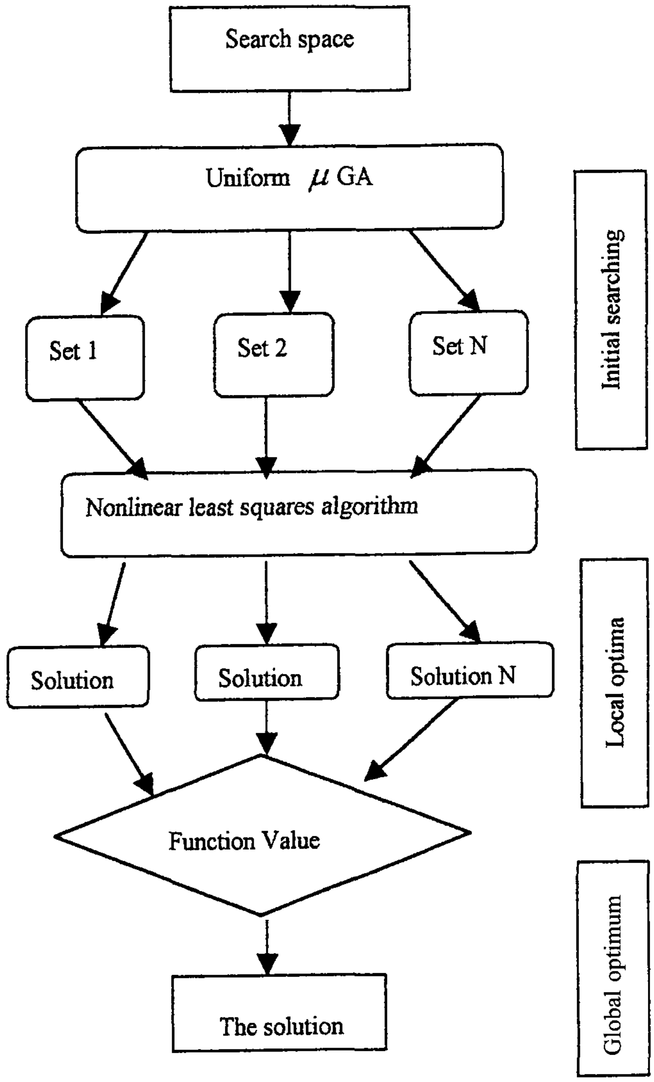

Figure 2 : Flow chart of the combined method

An NN model, which consists of a set of nodes arranged into layers, is shown in Figure 3. There are N inputs representing the responses on the surface and M outputs representing the parameters to be characterized. $W=\{w_{ij}^k, i=1,\cdots,N_i, j=1,\cdots,N_j, k=1,2,3\}$ is a matrix of weights corresponding to the connections between the layers. Where N_i and N_j are the $i^{th}$and j^{th} numbers of neurons for the ith and jth layers, respectively. Training of the NN model is referred to as the calculation of the weight matrix W using training data sets. Once the training is complete, the NN calculation is relatively fast and repeatable regardless of the complexity of the actual physics of the problem.

5 Applications

5.1 LSM for SiCC FGM Plate

The SiC-C FGM plate has been developed by combining materials of SiC and C using a chemical vapor deposition (CVD) technique. The material properties of the SiC and C monolith are given in Table 1 [24]. Young's modulus E, shear modulus G and Poisson's ratio ν of the SiC-C FGM plate are listed in Figure 4. In this figure, all the physics parameters are shown in dimensionless, except the Poisson's ration. The distribution of the content of SiC and C is obtained by following equation:

$$\rho = \rho_c V_c + \rho_{sic} V_{sic} \tag{8}$$

where ρ_c and V_c are the density and the volume fraction of the C monolith, respectively and ρ_{sic} and V_{sic} are those of the SiC monolith, respectively.

The SiCC FGM plate is divided into 5 layer elements; each layer element is modeled as an isotropic material for which the constitutive equations involve three independent material properties (E, ρ, ν). The true material properties are shown in Table 2. It can be found from Table 2 that the Poisson's ratio can be taken as a constant in each layer element. The Young's modulus

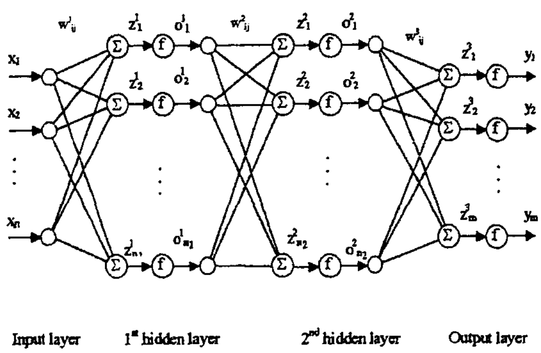

Figure 3 : A two-hidden layer NN model for material characterization of FGM plate.

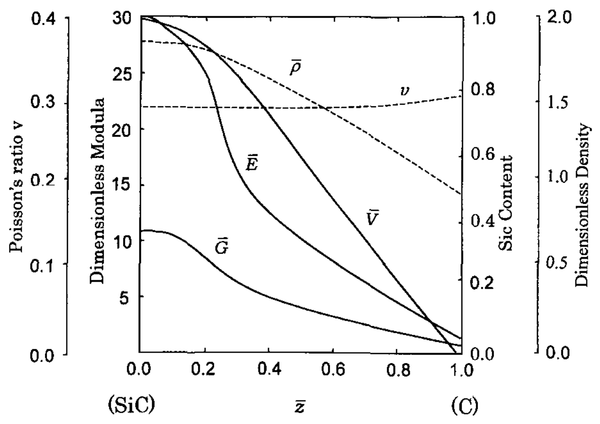

Figure 4 : Distribution of material property along the thickness direction of a SiC-C FGM plate

Table 2 : The true material properties of the SiC-C FGM plate

$\bar{z}$	ρ (g/cm^3)	$E(GPa)$	ν
0.00	1.78000	25.32438	0.30
0.10	1.79424	28.11006	0.30
0.20	1.83696	36.46710	0.30
0.30	1.90816	50.39551	0.30
0.40	2.00784	69.89528	0.30
0.50	2.13600	94.96641	0.30
0.60	2.29264	125.60890	0.30
0.70	2.47776	161.82276	0.30
0.80	2.69136	203.60798	0.30
0.90	2.93344	250.96456	0.30
1.00	3.20400	303.89251	0.30

Table 3 : The reconstructed results of the distribution of E and ρ of the SiC-C FGM plate

Coefficients	True value	Initial guess	Reconstructed result
A_E	2.62	3.14	2.62
B_E	0.00	0.00	0.00
C_E	28.78	34.53	28.77
A_ρ	1. 80	2.16	1.80
B_ρ	-0.01	0.01	-0.01
C_ρ	1.42	1.15	1.39

and mass density can be approximated by quadratic functions of thickness coordinate using the data set in Table 2 and a least square method. The functions are obtained as

$$\bar{E} = A_E + B_E\bar{z} + C_E\bar{z}^2 \tag{9}$$

$$\rho = A_\rho + B_\rho\bar{z} + C_\rho\bar{z}^2 \tag{10}$$

where $A_E = 2.62$, $B_E = 0.00$, $C_E = 28.78$; $A_\rho = 1.80$, $B_\rho = -0.01$, $C_\rho = 1.42$ and, $\bar{E} = E/const$, and $const = 9.68GPa$

Using LSM, E and ρ the as a function of the thickness are reconstructed, and listed in Table 3. It can be seen that the results are in very good agreement with the actual values. For this problem, it is found that the reconstructed material properties are not affected by the initial guesses if it is not exceed $\pm 40\%$ from the true values.

5.2 GA for SS-SN plate

SS-SN FGM is composed of stainless steel and silicon nitride; the material properties for stainless steel and silicon nitride are listed in Table 4 [25]. The silicon nitride is considered as the inclusion material.

Consider an FGM plate with varying material properties in the thickness direction. The material property of the FGM can be obtained using methods of rule-of-mixture [26] derived from the micro-mechanics using the material properties of the matrix and inclusion for given volume fractions.

The volume fractions in each elements surface are chosen as the parameters to be reconstructed for characterizing the material properties of FGM. If the plate is divided into m layer elements,

Table 4 : Material properties of stainless steel and silicon nitride monolith materials

Stainless steel			Silicon nitride		
$E\,(GPa)$	ν	$\rho(kg/m^3)$	$E\,(GPa)$	ν	$\rho(kg/m^3)$
207.82	0.3177	8166	322.4	0.24	2370

then there are a total of $m+1$ discrete value of volume fractions at the element interfaces and the upper and lower surfaces of the plate.

The plate is divided into 6 elements. It is assumed that this SS-SN FGM plate is made in such a way that the upper surface is pure silicon nitride and lower surface is pure stainless steel. Therefore, the volume fractions on the upper and lower surfaces are known as 1.0 and 0.0, respectively. Hence, there are five parameters in total need to be characterized. The original values of the volume fractions are assumed as the following function:

$$V_P = 1 - \bar{z}^3 \tag{11}$$

where $\bar{z} = z/H$. The bounds on the 5 parameters are set $\pm 30\%$ off from the actual value.

Characterization is based on an over-determined data set, which can be the time history of the displacement response at one receiver point on the upper surface of the SS-SN FGM plate. In a GA run, each individual chromosome represents a candidate combination of reconstructed parameters. For each candidate combination, forward calculation has to be performed to obtain u_i^c, the displacement response. These calculated displacement readings are used to obtain the fitness value of the candidate combination. The fitness value, which is defined using Eq. (2), will determine the probability of the candidate being chosen as a future parent. A FORTRAN sub-program for forward calculation using modified HNM is developed and interfaced with the GA main program.

To determine the volume fractions, a uniform μGA with binary parameter coding, tournament selection, uniform crossover and elitism is adopted as the inverse operator. Knuth's subtractive method is used to generate random numbers; Knuth's algorithm is regarded as one of the best random number generators. With a different negative number initialization, the Knuth's algorithm generates different series of random number. Elitism operator is adopted to replicate the best individual of current generation into next generation. The population convergence criterion is 5%, which means when less than 5 % of the total bits of other individuals in a generation are different from the best individual, the convergence occurs. The stopping criterion is imposed to limit each GA run to a maximum number of generations (say 500). The population size of each generation is set to 5.

Both the noise free and noise contaminated displacement response are used for the characterization of the volume fractions. The Gauss noise of various levels is directly to the computer-generated displacements. A vector of pseudo-random number is generated from a Gauss distribution with mean a and standard deviation b using Box-Muller method [27]. In this work, the mean a is set to zero, and the standard deviation b is defined as:

$$b = p \times [1/M(\sum_{i=1}^{M} u_i^m)^2]^{0.5} \tag{12}$$

Table 5 : Characterized volume fractions for SS-SN FGM plate

Position $\bar{z}$	Volume fractions	Original data	Results for different noise levels noise free	10% noise
0.3	V_P^1	0.973	0.967	0.976
0.5	V_P^2	0.875	0.860	0.863
0.7	V_P^3	0.657	0.656	0.676
0.8	V_P^4	0.488	0.488	0.500
0.9	V_P^5	0.271	0.276	0.253

Table 6 : Uniform μGA search space for glass/epoxy laminate

Parameter	True Data	Search Range (GA)	Possibilities #
E_1 (GPa)	38.48	19.00 - 58.00	1024
E_2 (GPa)	9.38	4.70 - 14.00	512
G_{12} (GPa)	3.41	1.70 - 6.00	512
ν_{12}	0.292	0.100 - 0.481	128
ν_{23}	0.507	0.250 - 0.760	128

whereis u_i^m the computer-generated displacements reading at the ith sample point, p is the value to control different levels of the noise, e.g. $p=0.05$ means 5% noise. The characterized results based on noise-free and Gauss noise-contaminated input data with a noise level of 10% are listed in Table 5. The present inverse procedure gives very accurate results as shown in Table 5. It can also be noted that the characterized results are very stable regardless of the levels of noise, even for the noise level of 10%.

5.3 GA for laminated Plate

Following the same way as Section 5.2, another example for determination of the material constant of laminated plate will be presented by means of the uniform μGA.

Consider a laminated plate consisting of twelve glass/epoxy layers. The stacking sequence of the laminated layer is denoted by $[0/+45/-45/90/-45/+45]_s$, where the digital numbers stand for the angles of fiber-orientation of each ply to the x-axis. The subscript of 's' means that the plate is symmetrically stacked. Though here we use a symmetrical laminate for the numerical verification, the method is applicable to laminates with arbitrary layer stacking sequence.

The GA search space defined for this numerical example is listed in Table 6. The bounds on the five parameters, two Young's modulus, one shear modulus and two Poisson's ratios, are set to approximate ±50% of the expected original values, then the five parameters are descritized and translated into a chromosome of length 39. In the whole search space, there are approximately 2^{39} possible combinations of the five parameters.

Eight GA runs with different random number series generated by Knuth's algorithm are performed and the mean values of identification results are shown in Table 7. It is found from tables that the identified results of young's modulus as well as shear modules are accurate and more stable to the added noise; while the results of Poisson's ratio have a comparable large variation when the noised is added. In general the results obtained are satisfactory.

Table 7 : Identification results for glass/epoxy laminate $[G0/45/-45/90/-45/45]_s$

Material Constant	True Data	Noise free Mean	% error	Noise(10%) Mean	% error
E_1 (GPa)	38.48	38.34	0.03	37.75	1.89
E_2 (GPa)	9.38	9.24	1.49	9.41	0.32
G_{12} (GPa)	3.41	3.48	2.05	3.31	2.93
ν_{12}	0.292	0.293	0.34	0.301	3.08
ν_{23}	0.507	0.495	0.39	0.479	5.52

Table 8 : Results from the nonlinear LSM for material characterization of glass/epoxy laminated plate

	GA result (Initial value) (1)	(2)	(3)	LSM result (1)	(2)	(3)
	39.7	38.9	42.8	39.6	**38.9**	37.96
	9.7	9.65	8.09	9.64	**9.61**	8.82
	3.2	3.2	3.79	3.19	**3.205**	3.38
	0.36	0.31	0.12	0.358	**0.305**	0.15
	0.5	0.503	0.31	0.497	**0.504**	0.396
error	5.3e-4	1.5e-4	1.2e-2	6.8e-7	**7.0e-8**	5.6e-7

The total computation time of the GA run heavily depends on the single run time of the forward calculation subprogram since any time increase for forward calculation will be amplified by 3001 times. Only high computation efficient forward approach can lead to a tolerable level of the computer time for each GA run. The HNM method has outstanding computation efficiency for the wave analysis of composite laminate.

5.4 Combined method for Laminated Plates

Consider the same problem as solved in Section 5.3, now the combined method is employed.

First, three sets of the parameters are selected based on the results generated from the 50 generations of the uniform μGA. The GA search space defined for this numerical example is same as shown in Table 6. The values of these selected sets of parameters are listed in Table 8, as the GA results. Then, we apply the nonlinear LSM three times using these better solutions as the initial points. These results are also shown in Table 8 as the LSM results. Finally, the global optimum can be found from these solutions by comparing error function values, as shown in Table 8 in bold fonts. The global optimum has the minimum value of the error function. There are in total 281 function evaluations needed in the present method, significantly less than the 3001 function evaluations required using the uniform GA alone for the same

Table 9 : Results for four-layer carbon/epoxy laminate at 300th generation using μGA and real-μGA

Material Constant	Actual Value	Search Result (μGA)	Search Result (Real-μGA)
E1(GPa)	142.17	138.0(-2.93%)	141.9(-0.19%)
E2(GPa)	9.255	9.294(+0.42%)	9.330(+0.81%)
G12(GPa)	4.795	4.779(-0.33%)	4.760(-0.72%)
γ_{12}	0.334	0.406(+21.56%)	0.342(+2.4%)
γ_{23}	0.486	0.462(-4.94%)	0.471(-3.09%)

Table 10 : Characterized volume fractions of an actual SiCC FGM plate

Position	Volume fractions	original data	Result (deviation)
(a) noise free			
$\bar{z}=0.2$	V_1	0.961	0.972(1.1%)
$\bar{z}=0.4$	V_2	0.842	0.835(-0.8%)
$\bar{z}=0.6$	V_3	0.634	0.632(-2.9%)
$\bar{z}=0.8$	V_4	0.367	0.385(5.0%)
(b) noise-added			
$\bar{z}=0.2$	V_1	0.961	0.969(0.8%)
$\bar{z}=0.4$	V_2	0.842	0.832(-1.2%)
$\bar{z}=0.6$	V_3	0.634	0.616(-2.8%)
$\bar{z}=0.8$	V_4	0.367	0.378(3.0%)

problem. The combination of two algorithms increases the efficiency greatly.

5.5 Real-μ GA for Carbon/epoxy Laminated Plate

Consider a composite laminated plate consisting of four carbon / epoxy layers. The stacking sequence of the laminated layer is denoted by [/0/+45/-45/0/]. The laminated plate is loaded by a half-cycle sine function dynamic load in z-direction over one element on the upper surface of the plate. Duration of the load is chosen as 1 millisecond. Both the loading location and receiving points are chosen arbitrarily. The displacement response in z direction at the receiving point is computed using LS_DYNA. The real-μ GA [17] is used in the material characterization of composite laminates for better solution at same generation. Results are given in Table 9.

5.6 NN model for SiC-C FGM

In this section, the NN model is used as the inverse procedure for material characterization. The actual SiC-C FGM plate in section 5.1 is still employed in this study to illustrate the application of NN model.

The modified hybrid numerical method (HNM) is employed as a teacher for the NN model training. The modified HNM allows a linear variation of material properties in the element in the thickness direction.

The material properties of the SiC and C monolith material are given in Table 1, the material C is regarded as the inclusion material. In this study, this SiC-C FGM plate is divided into five layered elements. It is assumed that this SiC-C FGM plate is made in such a way that the upper surface is pure carbon and lower surface is pure silicon carbon. Therefore, the volume fractions on the upper and lower surfaces are 1.0 and 0.0, respectively. Hence there are four parameters, named as v_1, v_2, v_3, v_4, need to be characterized.

Inputs for the NN model should be easily and accurate measured and are sensitive to the change of the volume fractions of the FGM plate. The displacement responses data on the surface of the FGM plate are selected as the inputs in this paper. As a necessary condition for successfully utilizing NN model, the sought outputs have to be dependent on the input data.

The set of the actual volume fractions for this SiC-C FGM is listed in the 3rd column in Table 10. An NN model is built for the characterizing the actual SiC-C FGM plate using training samples generated based on the range of actual volume fractions of this SiC-C FGM. The displacement responses of the sample points on the upper surface in the x direction of these two sets are calculated using modified HNM and used as inputs to the NN model. In order to simulate the measured displacement responses, noise-contaminated inputs are also used.

Table 10 summarizes the characterized result; it can be found that, the characterized result is very good, and the required number of progression is not changed, even when the noise is added.

6 Conclusion

Computational inverse techniques for material characterization of composites are proposed. In these techniques, the hybrid numerical method and finite element method is employed as the forward solver to calculate the dynamic displacement response on the surface of the composite plate for given material property. Some optimal and identification techniques, such as nonlinear least square method, genetic algorithm, method combining the GA with the nonlinear LSM, and NN model have been used as the inverse operator to determine the material property of composite plates.

We have demonstrated, through the use of the proposed computational inverse techniques for solving some practical problems of material characterization of composites, that material characterization of composite can be solved to a desired accuracy with high efficiency through innovative use of advanced computational techniques.

References

[1] Rokhlin, S.I. and W. Wang. Double Through-transmission Bulk Wave Method for Ultrasonic Phase Velocity Measurement and Determination of Elastic Constants of Composite Materials, The Journal of the Acoustical Society of America 91(6), pp. 3303-3312. (1992).

[2] Chu, Y.C. and S.I. Rokhlin. Stability of Determination of Composite Moduli from Velocity Data in Planes of Symmetry for Weak and Strong Anisotropies. The Journal of the Acoustical Society of America, 95(1), pp. 213-225. (1994).

[3] Chu Y.C. and S.I. Rokhlin. Analysis of Composite Elastic Constant Reconstruction from Ultrasonic Bulk Wave Velocity Data. Review of Progress in Quantitative Nondestructive Evaluation, edited by Thompson DO and Chimenti DE. Plenum Press, 13, pp.1165-1172. (1994).

[4] Bratton, R.L. and S.K. Datta. Anisotropic Effects on Lamb Waves in Composite Plates. Review of Progress in Quantitative Nondestructive Evaluation, edited by Thompson DO and Chimenti DE, Plenum Press, 8A, pp. 197-204. (1989).

[5] Rose, J.L., Huang, Y. and A. Tverdokhlebov. Surface Waves for Anisotropic Material Characterization: A Computer Aided Evaluation System. Review of Progress in Quantitative Nondestructive Evaluation, edited by Thompson DO and Chimenti DE, Plenum Press, 9. pp. 1573-1580. (1990).

[6] Nakamura, M. and K. Kimura. Elastic Constants of TiAl3 and $ZrAl_3$ Single Crystals, Journal of Material Science, 26, pp. 2208-2214. (1991).

[7] Sachse W. and K.Y. Kim. Point-source / Point-receiver Materials Testing. Review of Progress in Quantitative Nondestructive Evaluation, edited by Thompson DO and Chimenti DE, Plenum Press, 6A, pp. 311. (1986).

[8] Balasubramaniam, K. and N.S. Rao. Inversion of Composite Material Elastic Constants from Ultrasonic Bulk Wave Phase Velocity Data Using Genetic Algorithms. Composite Part B, 29B, pp.171-180. (1998).

[9] Rok Sribar. Solutions of Inverse Problems in Elastic Wave Propagation with Artificial Neural Networks. Dissertation, Cornell University, USA, (1994).

[10] Liu GR, Tani J, Ohyoshi T and Watanabe K. Transient waves in anisotropic laminated plates, Part 1: Theory; Part 2: Applications. Journal of vibration and acoustics. 113: 230-239, (1991).

[11] Liu, G.R., Tani, J., and Ohyoshi, T., "Lamb waves in a functionally gradient material plates and its transient response. Part 1 and Part 2," Trans. of the Japan society of mechanical engineers. Vol.57 (A), No. 535, pp.131-142, (1991).

[12] Liu, G.R., Chen, S.C., "Flaw detection in sandwich plates based on time-harmonic response using genetic algorithm," Comput. Methods Appl. Mech. Engrg., Vol.190, pp.5505-5514, (2001).

[13] Liu, G.R., Han, X., Lam, K.Y., "Material Characterization of FGM plates using elastic waves and inverse procedure," Journal of Comp. Mat.," Vol.35, No. 11, pp.954-971, (2001).

[14] Han X., Liu G.R., K.Y. Lam and T. Ohyoshi, "A quadratic layer element for analyzing stress waves in functionally graded materials and its application for material characterization. Journal of Sound and Vibration, Vol.236, No. 2, pp. 307-321, 2000.

[15] Liu, G.R., Ma W.B. and Han X., "An inverse procedure for determination of material constants of composite laminates," C Comput. Methods Appl. Mech. Engrg., to be published, (2001).

[16] Liu, G.R., Han, X., Lam, K.Y., "A combined genetic algorithm and nonlinear least squares method for material characterization using elastic waves," Comput. Methods Appl. Mech. Engrg. 191, 1909-1921, (2002).

[17] Liu, G. R., Ma, H. J., Wu, Z. P., "Material Characterization of Composite Laminate Using Dynamic Response and Real Parameter Coded Micro Genetic Algorithm," Submitted, (2001).

[18] Liu, G.R., Han, X., Xu, Y.G. and K.Y. Lam, "Material Characterization of Functionally Graded Material by Means of Elastics waves and a Prograssive-learning Neural Network," Composites Science and Technology, 61, pp. 1401-1411, (2001).

[19] Han, X. Liu G.R. and K.Y. Lam, "Determination of elastic constants of anisotropic laminated plates using elastic waves and a progressive neural network." Journal of Sound and Vibration, in press, (2001).

[20] Liu G.R., Lam K.Y. and Ohyoshi T. "A technique for analyzing elastodynamic responses of anisotropic laminated plates to line loads". Composites Part B; 28B: 667-677, (1997).

[21] Liu G.R., Lam K.Y. and Shang H.M., A new method for analyzing wave fields in aminated composite plates:
Two-dimensional cases. Composite Engineering. 5(12): 1489-1498, (1995).

[22] Philip, E. Gill, Walter, Murray, 1978 SIAM Journal for numerical analysis 15(5) Algorithms for the solution of the nonlinear least squares problem.

[23] Krishnakumar, K., "Micro-Genetic Algorithms for stationary and non-stationary function optimization," SPIE: intelligent control and adaptive systems, Vol.1196, Philadelphia, PA, (1989).

[24] M. Sasaki, Y. Wang, T. Hirano and T. Hirai Design of SiC/C functionally gradient materials and its propagation by chemical vapor deposition. Journal of ceramic society of Japan. 97(5), 539-543, (1989)

[25] Touloukian, Y.S., "Thermo-physical properties of high temperature solid materials," New York: Macmillan, (1967).

[26] Liu G.R., A step-by-step method of rule-of-mixture of fiber- and particle-reinforced composite materials. Composite structures. 40: 313-322. (1998).

[27] Kendall MG and Stuart A. The advanced theory of statistics (Vol.1). 3rd Edition. Griffin, London. (1969).