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Another Explanation of Our Contribution Function (*CFI*) by Our previously proposed General Perturbation Method for the Observed Perturbation of Intensity

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ABSTRACT

We try to explain validity of our previously proposed Contribution Function (hereafter,*CFI*) (Y.Ito,1988) by our Perturbation Method for observed Intensity *I (0)*, (Y.Ito,1997), which was also previously proposed in order to get the exact and general solution of observed perturbation of intensity, $\delta I(0)$. Our Contribution Function(*CFI*) has important essential problem, that is, how to understand the significance of integral of our Contribution Function(*CFI*) being not equal to observed Intensity *I (0)*, pointed out by the late Professor emeritus Juichi Hitotsuyanagi(1987). In later paper (Y.Ito,2003), to explain this we define new ContributionFunction (hereafter,*CFII*), and we have shown integral of new Contribution Function(*CFII*) is capable of being not equal to observed Intensity *I (0)*. But, *CFII* is slightly different with *CFI*. So, we must re-examine this problem, concerning our original Contribution Function(*CFI*). This is another problem of this paper.
Key words: line formation: observed intensity, contribution function,
--solar atmosphere

I. INTRODUCTION

In the former paper (Y.Ito,1988), (hereafter, Paper I), we have proposed new definition of Contribution Function as expressed in intensity unit, in order to solve the complicated problem of estimating the effective depth of the formation of the solar spectra.

In this paper, we try to explain this original Contribution function *(CFI)* by anothor plain method by using our Perturbation Method for observed Intensity *I (0)*, (Y.Ito,1997) (hereafter, Paper II), which was also previously proposed in order to get the exact and general solution of observed perturbation of Intensity, $\delta I(0)$.

Our Contribution Function(*CFI*) has important essential problem, that is, how to understand the significance of integral of our Contribution Function(*CFI*) being not equal to observed Intensity *I (0)*, pointed out by the late Professor emeritus Juichi Hitotsuyanagi(1987). In the recent paper we have tried to solve this essential problem (Y.Ito,2003) (hereafter, Paper III). This tryal was concerned with anoter defined Contribution Function(*CFII*) that is contribution of perturbation at a layer (z) to observed perturbation of intensity, $\delta I(0)$, which was defined from a view point of the above mentioned our general Perturbation Method for observed Intensity *I (0)*. But this *CFII* is slightly different with our original Contribution Function *CFI*. So, concerning *CFI*, we must re-examine this problem. This is another problem of this paper.

For this purpose, firstly, in the Chapter II we review the essence of the idea of our original Contribution Function *CFI* (Paper I). Secondly, in the Chapter III we try to show another explanation of our original Contribution Function *CFI* (Paper I) by another plain method by using above mentioned our Perturbation Method for observed Intensity $I(0)$ (Paper II). And, in the Chapter IV we shall show an "Experiment of Thought" concerning simple model atmosphere of the sun, for to get insight into *CFI*. In the Chapter V, we try to explain above mentioned essential important problem concerning *CFI*, pointed out by the late Professor emeritus Juichi Hitotsuyanagi(1987). Finally, in the Chapter VI, we shall make a few points about the results of this paper.

Ⅱ. The Essence of the Idea of our Contribution Function *CFI*

In the Paper I(1988), we defined new Contribution Function(*CFI*). The essence of the idea of our original Contribution Function *CFI* is, as follows:

If we confined attention exclusively to the planar geometry and the case $\theta = 0$, the transfer of equation of Intensity $(I(z))$is, at a given frequency,

$$\frac{dI(z)}{dz} = k(z)I(z) - j(z) \tag{1}$$

where k = the absorption coefficient per unit volume, j = the emission coefficient per unit volume, z = the geometrical depth along the line of sight (positive downward into the atmosphere). This equation means "local net loss, expressed in Intensity unit, at a layer (z)".

And, our original Contribution Function(*CFI*) have been defined, in a word, "the transfer of local netgain, expressed in Intensity unit, at a layer (z) to the surface", that is,

$$-\frac{dI(z)}{dz}e^{-\tau} = (j - kI)e^{-\tau} \tag{2}$$

where $\tau = \int_0^z kdz$.

Therefore, our original Contribution Function(*CFI*) of a layer(z) or layer(τ) are defined, respectively,

$$CFI(z) \equiv (j - kI)e^{-\tau} \tag{3}$$

$$CFI(\tau) \equiv (S - I)e^{-\tau}. \tag{4}$$

Where S is Source Function $= (j/k)$.

This difinition allow us to use, not only in the case of emission (for example, the case of solar atmosphere) but also in the case of no emission (for example, the case of gas cloud, the case of terrestrial atmosphere and so on). Many other Contribution Functions never can use in the case of no emission.

Ⅲ. Anothor Explanation of Our Original Contribution Function (*CFI*) by General Perturbation Method for the Observed Perturbation of Intensity

1. Derivation of Intensity Perturbation of $\delta I(t_0)$

In the former paper (Paper II), we assume that the perturbation can be put in the form of perturbation of

$j = \delta j(z)$ and perturbation of $k = \delta k(z)$, that is, j is changed into $j' = j + \delta j$ and k is changed into $k' = k + \delta k$, in all the layer of atmosphere of which we take notice. And we assume that these perturbations also produce the perturbation of Intensity $I = \delta I(z)$, that is, I is changed into $I' = I + \delta I$, where we call j' = perturbed emission coefficient, k' = perturbed absorption coefficient, I' = perturbed Intensity. Here, we need to remark on the point that the meaning of "perturbation" in our case is expanded to be applicable to not only in the case of weak perturbation but also in the case of "strong perturbation", that is, $\delta k/k \lll 1$, $\delta j/j \lll 1$.

And, we have got the general solution of this equation at a layer (z),

$$\delta I(z) \equiv e^{\tau(z)} \times e^{\delta\tau(z)} \int_z^{\infty} (\delta j - \delta k I) e^{-\tau} \times e^{-\delta\tau} dz . \tag{5}$$

In this paper, we use not *z-scale* but use t_0*-scale* or *x-scale*, so we must transfer Eq.(5) to t_0*-scale* or *x-scale*, where, $t_0 = \int_0^z k_{t0} dz$, k_{t0} = absorption coeficient per unit volume at line center. $x = \log\ t_0$.

From Eq.(5), if we assume $j_\ell = k_\ell$ we can derivate next equation.

$$\begin{aligned} \delta I(t_0) &\equiv I'(t_0) - I(t_0) \\ &= e^{\tau(t_0)} \times e^{\delta\tau(t_0)} \int_{t_0}^{\infty} \{(\delta\varphi_l + \delta r_0)S + [(\delta\varphi_l + r_0) + (\varphi_l + r_0)]\delta S - (\delta\varphi_l + r_0)I\} e^{-\tau(t_0)} \times e^{\delta\tau(t_0)} dt_0 . \end{aligned} \tag{6}$$

Where, $\phi_l = \dfrac{k_l}{k_{t0}} = e^{-x_{obs}{}^2}$, $x_{obs} = \dfrac{\Delta\nu}{\Delta\nu_D}$, $\Delta\nu_D$ = doppler half -width, $r_0 = \dfrac{k_c}{k_{t0}}$, k_l = line absorption coefficient per unit volume, j_l = line emission coficient per unit volume, k_c = continuam absorpion coefficinent per unit volume.

2. The Meaning of *CFI* $(I(t_0))$

In the Eq.(6), if we consider perturebed Intensity $\delta I(t_0)$, by the perturbation of Sorce Function from $S(t_0)$ to $I(t_0)$, so $\delta S(t_0) = I(t_0) - S(t_0)$, (no perturbation of φ_l and r_0), we can get,

$$\delta I(t_0) \equiv I'(t_0) - I(t_0) = e^{\tau(t_0)} \int_{t_0}^{\infty} (\varphi_l + r_0) \delta S(t_0) e^{-\tau(t_0)} dt_0 . \tag{7}$$

So, from Eq.(7), we can define our original *CFI(I (t0))*, by another way,

$$CFI(I(t_0)) \equiv \frac{d[\delta I(t_0) e^{-\tau(t_0)}]}{dt_0} = (\varphi_l + r_0)(S(t_0) - I(t_0)) e^{-\tau(t_0)} . \tag{8}$$

This definition of $CFI(I(t_0))$ means "differentiation at a layer (t_0) of transfer of Intensity perturebion $\delta I(t_0)$ to the surface" or "the transfer of local netgain, expressed in Intensity unit, at a layer (t_0) to the surface".

3. The Explanation of *CFI* $(I(t_0))$ by The Photon Number Counts Transfer.

For the sake of understanding the meaning of our original *CFI(I (t0))* more direct way,we use the Photon Number Counts, *Pc(t0)*, *Pgain(t0)*, where $Pc(t_0) \equiv \dfrac{I(t_0)}{h\nu}$, $Pgain(t_0) \equiv \dfrac{S(t_0)}{h\nu}$.

So, we can get,

$$Pc(t_0) = e^{\tau(t_0)} \int_{t_0}^{\infty} Pgain(t_0) e^{-\tau(t_0)} dt_0 . \tag{9}$$

And we define, $Pc2(t_0) \equiv \frac{I'(t_0)}{h\nu}$, $Ploss(t_0) \equiv \frac{I(t_0)}{h\nu}$, where $I(t_0) \equiv e^{\tau(t_0)}\int_{t_0}^{\infty} I(t_0)e^{-\tau(t_0)}dt_0$.

So,we can get,

$$Pc2(t_0) = e^{\tau(t_0)}\int_{t_0}^{\infty}(\varphi_l + r_0)Ploss(t_0)e^{-\tau(t_0)}dt_0 \ . \tag{10}$$

And,

$$\delta Pc(t_0) \equiv Pc(t_0) - Pc2(t_0) = e^{\tau(t_0)}\int_{t_0}^{\infty}(\varphi_l + r_0)Pnetgain(t_0)e^{-\tau(t_0)}dt_0 \ , \tag{11}$$

where, $Pnetgain(t_0) \equiv \frac{S(t_0) - I(t_0)}{h\nu}$

So, we can define our original *CFI (Pc (t0))* by direct way,

$$CFI(Pc(t_0)) \equiv -\frac{d[\delta Pc(t_0)e^{-\tau(t_0)}]}{dt_0} = (\varphi_l + r_0)Pnetgain(t_0)e^{-\tau(t_0)} \ . \tag{12}$$

This definition of *CFI (Pc(t0))* means "differentiation at a layer (t_0) of transfer of Photon Number Counts perturbation $\delta Pc(t0)$ to the surface" or "the transfer of local netgain, expressed in Photon Number Counts unit, at a layer (t_0) to the surface".

Ⅳ. Example of application of our original *CFI*

1. The Case of the Solar Atmosphere

For, getting insight of *CFI*, we show an " Experiment of Thought " concerning simple model atmosphere of the sun.

For the sake of convenience, we use next normalyzed notations.

$$\left.\begin{aligned}
&pc(t_0) \equiv Pc(t_0)\frac{h\nu}{B_0} = \frac{I(t_0)}{B_0}, \quad pgain(t_0) \equiv Pgain(t_0)\frac{h\nu}{B_0} = \frac{S(t_0)}{B_0}, \\
&pc2(t_0) \equiv Pc2(t_0)\frac{h\nu}{B_0} = \frac{I'(t_0)}{B_0}, \quad ploss(t_0) \equiv Ploss(t_0)\frac{h\nu}{B_0} = \frac{I(t_0)}{B_0}, \\
&pnetgain(t_0) \equiv Pnetgain(t_0)\frac{h\nu}{B_0} = \frac{S(t_0) - I(t_0)}{B_0}, \\
&\text{and, we assume } S(t_0) = B_0(1 + \beta t_0).
\end{aligned}\right\} \tag{13}$$

If, we use $x = log\ t_0$, for depth scale, we get,

$$pgain(x) = (1 + \beta 10^x)\frac{10^x}{\log e} \ , \tag{14}$$

$$pc(x, x_{obs}) = e^{\tau(x)}\int_x^{\infty}(\varphi_l + r_0)pgain(x)e^{-\tau(x)}dx = (1 + \beta 10^x) + \frac{\beta}{\varphi_l + r_0} \ , \tag{15}$$

$$ploss(x,x_{obs}) = pc(x,x_{obs}) \times \frac{10^x}{\log e} = [(1+\beta 10^x) + \frac{\beta}{\varphi_l + r_0}] \times \frac{10^x}{\log e} \quad , \tag{16}$$

$$pc2(x,x_{obs}) = [(1+\beta 10^x) + \frac{\beta}{\varphi_l + r_0}] + \frac{\beta}{\varphi_l + r_0} \quad , \tag{17}$$

$$pnetgain(x,x_{obs}) = pgain(x) - ploss(x,x_{obs}) = -\frac{\beta}{\varphi_l + r_0}\frac{10^x}{\log e} \quad , \tag{18}$$

$$\delta pc(x,x_{obs}) \equiv pc(x,x_{obs}) - pc2(x,x_{obs})$$
$$= e^{\tau(x)} \int_x^{\infty} (\varphi_l + r_0)\, pnetgain(x,x_{obs}) e^{-\tau(x)} d_x = -\frac{\beta}{\varphi_l + r_0} \quad . \tag{19}$$

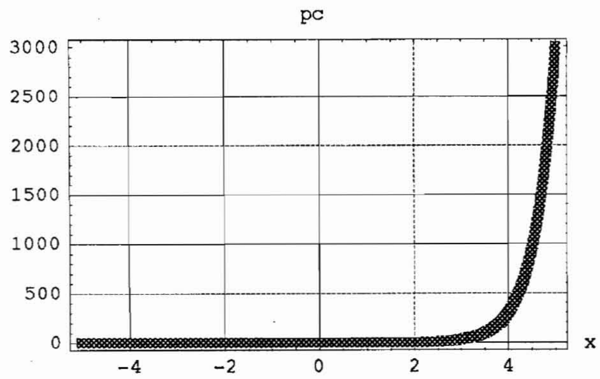

Fig.1. pc (x) ($x_{obs}=0$)

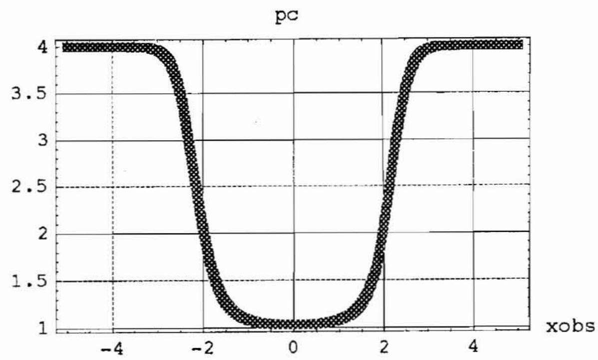

Fig.2. pc (x_{obs}) ($x=-\infty$)

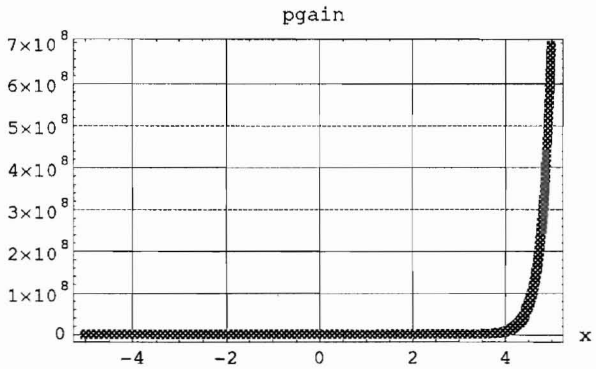

Fig.3. pgain (x)

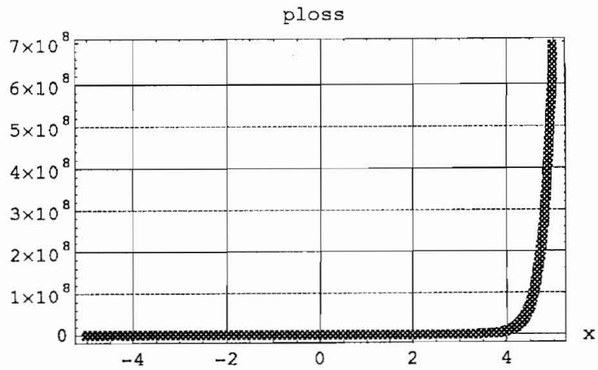

Fig.4. ploss (x, x_{obs}) ($x_{obs}=0$)

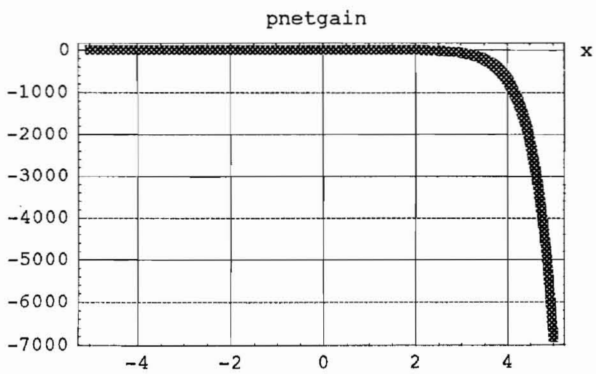

Fig.5. pnetgain (x, x_{obs}) ($x_{obs}=0$)

So, we can define *CFI (pc (x,xobs))*,

$$CFI(pc(x,x_{obs})) \equiv -\frac{d[\delta pc(x,x_{obs})e^{-\tau(x)}]}{dx}$$

$$= (\varphi_e + r_0)pnetgain(x,x_{obs})e^{-\tau(x)} = -\beta e^{-\tau(x,x_{obs})}\frac{10^x}{\log e}, \qquad (20)$$

where $\tau(x, x_{obs}) = (\varphi_e(x_{obs}) + r_0)10^x$.

This definition of *CFI (pc(x,xobs)))* means "differentiation at a layer (*x*) of transfer of photon number counts perturbation δpc *(x,xobs))* to the surface".

2. The meanig of *CFI (pc* $(x,x_{obs})))$

The meanig of *CFI* $(pc(x,x_{obs})))$ can expressed more simple than above expression, that is, "transfer of netgain of photon number counts (=photon gain -photon loss) at layer $(x,\ x+dx)$ to the surface".

We can show the effective depths of formation of solar line spectrum. For line center, *xobs*=*0*, from Fig 6, absolute maximum value of *CFI* $(pc(x,x_{obs}))$ is at $x=0$, so, the effective depth of the formation $x=0$. For line wing, *xobs*=*5*, from Fig 7, absolute maximum value of *CFI* $(pc(x,x_{obs}))$ is at $x=2$, so,the effective depth of the formation $x=2$. These are reasonable depths.

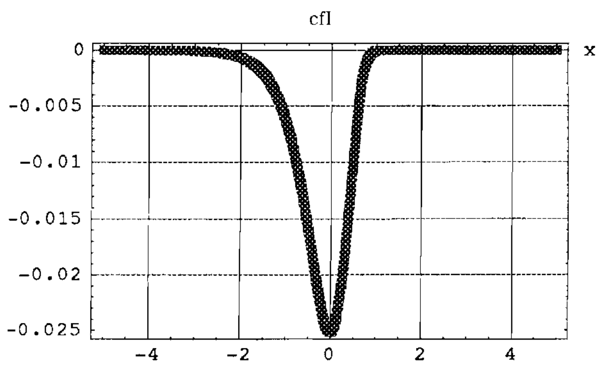

Fig.6. CFI $(pc(x,\ x_{obs}))$ $(x_{obs}=0)$

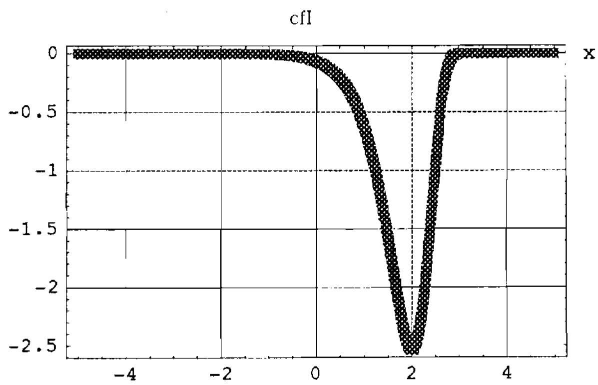

Fig.7. CFI $(pc(x,\ x_{obs}))$ $(x_{obs}=5)$

3. The meanig of *CFI* $(pc(x,x_{obs})))$ having minus value

From Eq.(18) and Eq.(20), "netgain of photon number counts at layer $(x,\ x+dx)$ "$=(\varphi_l+r_0)$ *pnetgain* $(x,\ x_{obs})$ and "transfer of netgain of photon number counts at layer $(x,\ x+dx)$ to the surface"=*CFI(pc* $(x,\ x_{obs}))$ both have minus value. So, a doubt may be raised, that is "transfer of minus value of netgain of photon number counts at layer (*x*) to the surface" has no physical means. But, we can explain that (φ_l+r_0) *pnetgain* $(x,\ x_{obs})$ or *CFI* $(pc(x,\ x_{obs})))$ has the meaning against plus value $pc(x,\ x_{obs})$ or transfer of $pc(x,\ x_{obs})$. So, minus value of *CFI* $(pc(x,\ x_{obs})))$ means "transfor of extinction of $pc(x,\ x_{obs}))$ at layer(*x*)" to the surface.

V. The Explanation of The Essential Probem of our original Contribution Function *CFI*, pointed out by the late Professor emeritus Juichi Hitotsuyanagi

Our Contribution Function (*CFI*) has above mentioned essential problem, pointed out by the late Professor emeritus Juichi Hitotsuyanagi (1987), that is, how to understand the significance of integral of our Contribution Function (*CFI*) being not equal to observed Intensity $I(0)$.

Namely, the problem is, as follows :

From Eq.(8), if we integrate $CFI(I(t_0))$, from $t_0 = 0$ to $t_0 = \infty$, we get

$$\int_0^\infty CFI(I(t_0))dt_0 = \int_0^\infty (\varphi_l + r_0)(S(t_0) - I(t_0))e^{-\tau(t_0)}dt_0 = I(0) - \int_0^\infty (\varphi_l + r_0)I(t_0)e^{-\tau(t_0)}dt_0 \quad (21)$$

For the case of solar atmosphere, from Eq.(15), Eq.(16), Eq.(18) and Eq.(20), if we integrate $CFI(pc(x))$, from $x = -\infty$ to $x = \infty$, we get

$$\int_{-\infty}^\infty CFI(pc(x, x_{obs}))dx = \int_{-\infty}^\infty (\varphi_l + r_0)(pgain(x) - ploss(x, x_{obs}))e^{-\tau(x)}dx$$
$$= pc(-\infty) - \int_{-\infty}^\infty (\varphi_l + r_0)pc(x, x_{obs})e^{-\tau(x)}\left(\frac{10^x}{\log e}\right)dx \quad (22)$$

At first sight, this result may seem to be questionable. Because second terms of Eq.(21) and Eq.(22) are like to be unnecesary. So we must explain the reason why we can adopt our Contribution Function *CFI*.

In the latest paper, we have tried to solve this essential problem (Paper III). This tryal was concerned with anoter defined Contribution Function(*CFII*) that is "contribution of perturbation at a layer (z) to observed perturbation of intensity, $\delta I(0)$ ". Namely, if we use t_0, for depth scale, we have cosidered observed Intensity perturbation, $\delta I(t_0)$, from the wavelength of the continuum, $I(0)$ to the wavelength of line, $I'(0)$, that is, perturbation of $\varphi_l = 0$ to $\varphi_l' = \varphi_l$ (so, $\delta\varphi_l = \varphi_l$). But this *CFII* is slightly different with *CFI*.

So, in this paper, we must positively explain the reason why we can adopt our original Contribution Function (*CFI*), inspite of the problem.

We show an explanation this problem as follows :

From Eq.(7) and Eq.(8), if we integrate $CFI(I(t0))$, from $t_0 = 0$ to $t_0 = \infty$, we get

$$\int_0^\infty CFI(I(t_0))dt_0 = -\delta I(0) = I(0) - I'(0) \quad (23)$$

For the case of solar atmosphere, from Eq.(19), Eq.(20), if we integrate $CFI(pc(x))$, from $x = -\infty$ to $x = \infty$, we get

$$\int_{-\infty}^\infty CFI(pc(x, x_{obs}))\,dx = \delta pc(-\infty) = pc(-\infty) - pc2(-\infty) = -\frac{\beta}{\varphi_l + r_0} \quad (24)$$

So,we can understand $CFI(I(t_0))$ is integrand of $(-\delta I(t_0))$, a kind of Response Function by the perturbation of Sorce Function from $S(t_0)$ to $I(t_0)$, (so $\delta S(t_0) = I(t_0) - S(t_0)$).

And, we can understand $CFI(pc(x, x_{obs}))$ is integrand of $\delta pc(-\infty)$, a kind of Response Function by the perturbation of Sorce Function from $pc(x, x_{obs})$ to $pgain(x)$, (so $\delta pgain(x) = pgain(x) - pc(x, x_{obs})$). (we defined general Response Function in the former paper (Paper II) by Eq.(IV.3).)

VI. Discussions

In conclusion, we make several points of this paper.

1. We try to explain varidity of our originaly proposed Contribution Function(*CFI*) (Paper I) from a view point of our Perturbation Method for observed Intensity $I(0)$, (Paper II), which was previously proposed in order to get the exact and general solution of observed perturbation of intensity, $\delta I(0)$. These essence of the idea are mentioned in Chapter II and Chapter III. *CFI(z)* have been defined, in a word, " the transfer of local net gain, expressed in intensity unit, at a layer (*z*) to the surface " .

2. For understing the meaning of *CFI*, we use the Photon Number Counts, *Pc(t0)*, *Pgain(t0)* et al.. And, in the case of solar atmosphere, for the sake of convenience, we use normalyzed photon number count, *pc(x)*, *pgain (x)* et al.. So, the meanig of *CFI(pc(x))* can expressed more simple than above expression, that is, "transfer of netgain of photon number counts at layer(x) to the surface".

3. But, in the case of solar atmosphere, " netgain of photon number counts at layer(x) "=*pnetgain(x, xobs)* and " transfer of netgain of photon number counts at layer(x) to the surface " = $CFI(pc(x, x_{obs}))$ both have minus value. We can explain that these minus values of *netgain(x, xobs)* and $CFI(pc(x, x_{obs}))$ have the meaning against plus value $pc(x, x_{obs})$. So minus value of $CFI(pc(x, x_{obs}))$ means " transer of extinction of $pc(x, x_{obs})$ at layer(x) to the surface " .

4.. In the later Paper III, we have shown integral of *CFII* is capable of being not equal to observed Intensity $I(0)$. But *CFII* is slightly different with *CFI*. So, in this paper we positively explain the reason why we can adopt our Contribution Function(*CFI*). We can understand $CFI(I(t_0))$, $CFI(pc(x))$ are respectively, integrand of Intensity perturbation, $(-\delta I(0))$, a kind of Response Function by the perturbation of Sorce Function from $S(t_0)$ to $I(t_0)$, so $S(t_0)=I(t_0)-S(t_0)$, and integrand of perturbation of photon number counts, $\delta pc(-\infty)$, a kind of Response Function by the perturbation of Sorce Function from $pc(x, x_{obs})$ to $pgain(x)$, so $\delta\, pgain(x, x_{obs})= pgain(x)-pc(x, x_{obs})$. So, integral of *CFI* is capable of being not equal to observed Intensity $I(0)$ or $pc(-\infty)$.

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