Combining Economic, Geological, and Technical Uncertainties in Mining Projects Valuation Using Real Options Analysis

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Abstract

The selling of examined metal is the only mining company revenue generator. It implies a certain amount of metal and selling price would grow their revenue. Before mining, the company should estimate the reserves with tremendous uncertainty. In addition, fluctuation of the metal price as economic uncertainty forces the company to deal with an advanced strategy to gain optimum value because profit is very sensitive to the selling price. Nevertheless, the cost of mining operations is unstable due to both local and global economic conditions. Cost instability represents the technical uncertainty of the process that consists of mining and processing-related expenditure.

Geology and mining operations need excessive capital expenditure to run the project. In contrast, finance has to press the expense to guarantee profit. A balance between them is the crucial success of mining. Exploration is necessary to estimate the resource. The estimation is detailed with core drilling at a considerable cost and yielded a certain confidence level of reserves with inherent uncertainty. Sales prices and expenses are commonly assumed in constant or constant growth without compromising their fluctuation. However, project evaluation cannot ignore those uncertainties to get the project value.

A standard method to evaluate a project value, Discounted Cash Flow (DCF), could not adequately account for the future risk. Net Present Value (NPV) as the decision parameter of the DCF method theoretically only generates two decision areas that are accepted and declined. While in reality, management commonly takes no action to wait for a reasonable commodity price, stable cost and ensure the number of reserves by collecting more exploration data. As a result, recent study dedicated to accounting for uncertainty, real options (RO) valuation, adapt financial option theory to be practiced in a real business. On the other hand, uncertainty in reserves is modelled by geostatistics methodology, kriging, and conditional simulation, which captures the spatial variability of the deposits.

There are three approaches in RO methodology: Black Scholes (BS) Valuation, Binomial Lattice (BL) Valuation, and simulation. The complexity of RO in BS and BL approach arises when considering multi uncertainties in project evaluation. On the other hand, simulation approach in RO is not well developed. As a result, RO studies often only consider price as an uncertainty driver. This research combined price, grade and cost uncertainty in a

mining project evaluation through the simulation approach, namely Multistage Stratified Stage Aggregation (MSSA), which would be the study's originality. Conditional simulation methods in geostatistics will be utilized to account for grade uncertainty. Thus, the expected reserve and the deviation are incorporated with commodity price and cost uncertainties.

This study demonstrated a project evaluation method covering resource estimation, mine planning, economic evaluation, and uncertainty assessment. The data was collected from PT Timah, Tbk, the most significant world tin producer in 2020, located in Indonesia. The data consisted of drill hole exploration and historical operation costs, while price data was recorded from the S&P 500. Those data were followed by resource estimation using conditional simulation, particularly Sequential Gaussian Simulation, which was run with the GEOVIA Surpac mining program. Mine planning and project evaluation were conducted with the GEOVIA Whittle mining program and converted to a monthly cash flow model. Finally, the uncertainty assessment was done with the real options method, especially MSSA, which utilized java programming language. The originality of the methodology was the development of path generation through java which is an essential step in the MSSA method.

Our research was a pilot method that demonstrated a combination of advances in resource estimation and economic evaluation. The conditional simulation method indicated that each reserve location had its geological uncertainty. Furthermore, Geometric Brownian Model (GBM) was used to make price simulations and get the price uncertainty. Lastly, we utilized the Mean Reverting Process to model cost uncertainty. The three uncertainties represented geological, economic, and technical uncertainties, respectively. MSSA is an alternative method to get project value considering those three uncertainties. A benchmark comparison of the MSSA result with BS and BL approaches resulted in a slight difference.

In summary, we developed a project evaluation methodology that considered geological, economic, and technical uncertainties represented by grade, price and cost, respectively. In our case study, the manager can run the project, but they must ensure the project cost. In addition, the price and geological uncertainties will not be revealed until they decide to mine its reserve; thus, controlling the production and price is essential to guarantee project profitability.

Keywords: project evaluation, real options analysis, uncertainty modeling, combining uncertainties, Multistage Stratified State Aggregation

Chapter 1 Introduction

1.1. Background

Standard mining project valuation starts from exploration to economic evaluation conducted by the company. The general purpose of exploration is to delineate valuable resources and estimate the reserve. Moreover, resources and reserves classification in exploration involves several branches of knowledge. Figure 1.1 depicts the framework for classifying mineral deposit to reflect varying levels of geological confidence and different degrees of technical and economic judgement, summarized as modified factors. Mineral resources can be estimated based on geoscientific data. Ore reserves, which are an altered selection of the indicated and measured mineral resources (shown within the dashed outline in Figure 1.1), must consider the modifying factors that affect extraction. It should be assessed with input from various disciplines. A competent person can transform measured mineral resources into either proven or probable ore reserves (The JORC Code 2012 Edition, 2012).

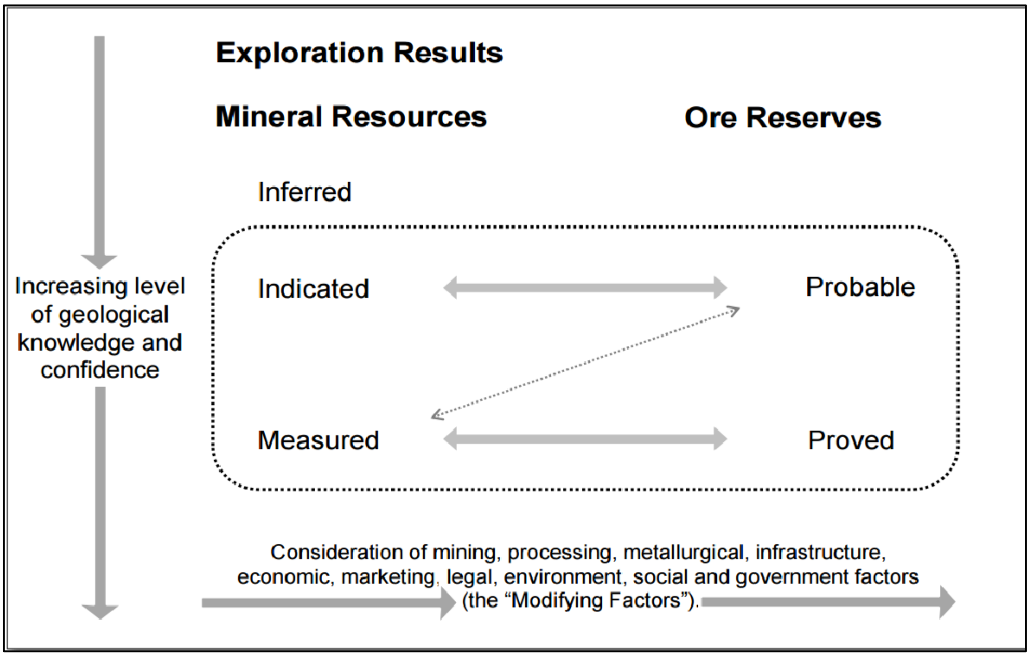

Figure 1.1. Resource and Reserve Classification

Source: JORC Code 2012 Edition, 2012

After exploration, a mining company obtains reserve estimation in a particular area with inherited uncertainty. Then, the reserve is evaluated in a feasibility study, including mine planning to extract the expected amount of metal. Once the reserve is extracted, the company invests in equipment, processing plant, and other expenditure. The investment should be assessed in detail, including the uncertainty. Practically, the evaluation method used by a company is the traditional Discounted Cash Flow (DCF) methodology. However, the method employed by common mining companies does not account for risk and uncertainty. Decision-makers frequently modify the discount rate to account for risk and integrate technical risk premium. Aside from that, they conducted a sensitivity analysis to forecast the impact of changing cash flow parameters. In some circumstances, this method might be a significant limitation in adoption; and an improper valuation technique (Nicholas, 2014).

Geology, mine plan, and finance departments forecast the DCF return of a typical mining project based on significant assumptions about price, reserves, and cost. These profit determinants come with risk and uncertainty. To date, the best method for modelling the resource beneath the surface is to use a coring drill. A significant geological uncertainty, grade uncertainty, will be assessed in this study through conditional simulation (CS) methodology. CS resulted in resource estimation and inherited geological uncertainty (Dowd, 1994). The CS method developed further kriging method, particularly generation of grade uncertainty. The study considered geological uncertainty which is typically not identified in economic evaluation instead of estimation.

With the confirmation of modifying factors, the resource was transformed into reserves. Operating costs, including mining, processing, and metallurgical operations, represent technical uncertainty. Furthermore, price uncertainty is a mining project's most significant profit determinant. Failure to respond to those risks and uncertainties will cause a significant mistake in evaluating the future worth of the project. Risk refers to the possibility of an undesirable occurrence occurring. In contrast, uncertainty refers to the future outlook of the project being unpredictable (Mun, 2006).

Researchers devised the Real Option (RO) to replace the DCF technique to account for risk and uncertainty. An option is a financial derivative whose value depends on the price of a stock. Option holders have the right but no obligation to execute their option by the maturity date. These alternatives are quantified by option calculation to justify option value. The adaptation of option calculation in the actual project is called real options (RO) (Hull, 2015).

To date, three approaches are used to calculate the value of an option: Black-Scholes Formulation, Lattice Valuation and Simulation Method. The Black Scholes Formula employed a closed mathematical approach (Black & Scholes, 1973). On the other hand, lattice valuation (Cox et al., 1979) used risk-neutral probability, which simplified the Black Scholes method's mathematical equation.

The simulation technique has similarities with lattice valuation. However, based on the Monte Carlo simulation methodology, the simulation method is designed to calculate high dimensional risk and uncertainty (Barraquand & Martineau, 1995; Hull, 2015). By optimizing the exercise and waiting for the value of the project, the lattice and simulation methods consider management flexibility. A temporal stochastic model and theoretical computation for natural resource project appraisal are introduced in the study of RO in natural resources (Brennan & Schwartz, 1985). Furthermore, the RO technique considers the managerial flexibility to amend a future choice if the ambiguity is exposed. The Black Scholes method (Black & Scholes, 1973) and lattice valuation (Cox et al., 1979), two common RO approaches, become complicated when accounting for more than one uncertainty. Nonetheless, the mathematical complexity (Haque et al., 2014) and usability (Ajak & Topal, 2015) of RO in the industry are still limiting factors. As a result, the simulation technique is vital to be developed further to solve the limitation.

The limitation of mathematical complexity was solving higher-order and dimensional partial differential equations. The shortage of empirical BS and BL approach was generally addressed by the simulation approach, Stratified State Aggregation (SSA). It was founded by Barraquand & Martineau (Barraquand & Martineau, 1995) for the American option and later developed by Adachi et al. for an oil industry (Adachi et al., 2008). However, mining is a unique industry that has complex uncertainties, and no detailed research has been developed. Therefore, we proposed an originality on evaluation method on a mining project considering grade, price and cost uncertainties through developed SSA approach.

A case study is conducted on tin mining projects at PT Timah Tbk, a mining company in Indonesia, particularly on monthly mine planning for underwater mining and using dredge as the primary equipment. Economic, geological, and technical factors of uncertainties are represented by price, grade, and cost, respectively. The application of RO in the mining industry could be improved by incorporating these uncertainties. However, modelling those uncertainties is challenging. Therefore, we proposed a modelling method to be applied in the mining industry. The model can be applied to other mining with adaptation to assumptions relating to mineral properties, mining methods and global economic conditions. In addition, CS methodology should increase RO application, particularly in analyzing geological uncertainties in a multi-stage of a short-term mining project. Multi-stage terminology refers to the total amount of mining locations as more than one. Our model covered resource estimation through the geostatistics method to mine plan on a monthly basis and economic valuation of two areas simultaneously. The option type is the waiting to operate option based on the data availability. The company has the right to operate, wait for the uncertainty to be resolved, or abandon the mining projects.

Our established valuation approach addressed limitations in RO applications, particularly concerning assessing several sources of uncertainty in a multi-stage short-term mining project. The application exhibited practical simplicity as well as applicability. We enhanced previous studies (Adachi et al., 2008) by employing the adaptation of spatial uncertainty through conditional simulation (CS) and the construct path generation algorithm to incorporate the three uncertainties. Closely related studies demonstrated the same using empirical models (Brennan & Schwartz, 1985; Cortazar et al., 2001). However, we utilized the simulation method (Barraquand & Martineau, 1995) and the uncertainty drivers. The methodology incorporated the economic impact of the reserve, pricing, and cost uncertainties. However, other modifying factors such as political, legal, social, and environmental are assumed to be stable.

1.2. Problem Statement

Metal prices have been fluctuating remarkably in recent years. Fluctuation of the price certainly affects mining company value. On the other hand, total available reserves are always uncertain until the deposit is depleted. The company should optimize the cost and obtain exploration data to estimate the reserves. Moreover, the history of mine operational cost indicated significant volatility that gives the most considerable risk to the tin mining company. These three uncertainties are calculated in this research through real options methodology, particularly Multi-stage SSA.

In this study, there are five main problems regarding measuring project value. As a case study, we demonstrated the case of the Indonesian tin company, PT Timah Tbk. They are as follows:

1. How do a mining company quantify geological uncertainty and use it to calculate project value?
2. How can a mining project value be calculated simultaneously considering economic, technical, and geological uncertainties?
3. How to evaluate the multi-project mining using RO methodology?
4. What is the advantage and disadvantage of the simulation method compared with Black-Scholes and lattice valuation?
5. What are the practical consequences of risk and uncertainty caused by price, cost, and reserve in the mining industry?

1.3. Purpose of Study

The mining industry faced uncertainties, particularly price fluctuation, the geological presence of the reserves, and technical-cost uncertainty. A developed evaluation method of a project is required considering the three most significant problems in mining. In addition, decision-making in a mining company practically needs coordination of geological, the finance and the mining engineering department. The finance department urges to decide the cheapest operating cost at the highest price. In contrast, the geological and mining engineering department wants to minimize the uncertainty of the existence of the ore and easy operation (Crundwell, 2009).

This research tried to meet their need using geostatistics and real options methodology. It resolved the price fluctuation that directly affected the project value and assessed the exploration step to account for grade volatility. Moreover, the fluctuation of the historical cost was measured through statistical method. The proposed method calculated the consequence of uncertainties to the project value. The assessment was conducted in a monthly time frame to describe how the real option worked.

We proposed an enhanced evaluation SSA method of mining projects considering their inherent uncertainties. The price and cost uncertainty was analyzed using its historical data with Geometric Brownian Model and Mean Reverting Process. On the other hand, geological uncertainty was explained through conditional simulation. Furthermore, we combined those uncertainties with the utilized SSA approach. The improvement of applicability of real options valuation and the utilization of conditional simulation to capture geological uncertainty was expected.

1.4. Significance of Study

To date, researchers developed methods to measure and mitigate the risk embedded in a business. They made parameters to calculate the return of a project and the inherited risk. Principally, a higher risk of a project should give the investor the best return. Furthermore, the most popular method was discounted cash flow analysis, which decided to either accept or decline to start. However, it had assumption that the risk would be constant during the project life. The developed risk assessment methodology, namely real options analysis, was adapted from financial option theory and practiced in real projects. It figured out the risk based on the potential growth of the project and the probability embedded.

The originality of proposed SSA methodology exhibits practical simplicity and enhances previous studies (Adachi et al., 2008) by employing the adaptation of spatial uncertainty through conditional simulation (CS) and the construction path generation algorithm to incorporate price and grade uncertainties. Closely related studies demonstrated using empirical models (Brennan & Schwartz, 1985; Cortazar et al., 2001). In contrast, we utilized the simulation method (Barraquand & Martineau, 1995) to assess the uncertainty drivers, which are more straightforward and flexible. Researchers concluded that the methodology is growing in metal mining due to increased uncertainty of markets and the complexity of new projects (Savolainen, 2016).

The uncertainties are assessed in short-term mining design or monthly mine planning incorporating economic, technology, and geological uncertainties. Additionally, the utilization of conditional simulation, which is typically an estimation tool, was first performed to assess grade uncertainty which would be the significance of this study. By examining the research, the project's decision-making was resulted in optimizing the project value. The methodology was a pilot demonstration to improve the applicability of RO methodology by capturing several uncertainties in a single evaluation. We developed the path generation in the MSSA method and applied specific model in uncertainties measurement. Nevertheless, technical issues such as geotechnical, weather, recovery, and other modifying factors (JORC, 2012) could be calculated in the future using RO Methodology, particularly simulation method with MSSA approach.

1.5. Outline of Study

The research on "Combining Economic, Geological, and Technical Uncertainties in Mining Projects Valuation Using Real Options Analysis" consisted of 5 chapters, including introduction (Chapter 1) and conclusion (Chapter 5).

Chapter 1 introduced the background of this research and set the purposes we intend to achieve. A brief explanation of recent studies regarding project evaluation, notably RO Analysis, and the gap of recent research is described in the chapter. In addition, we explained the significance of the studies that would be the originality and contribution to current research.

Chapter 2 consisted of explaining the case study using the typical traditional evaluation method. It covered exploration, mine planning, and project evaluation. Moreover, it contained collected data and how the project valuation was carried out. This chapter also assessed the positive and negative points of current practice. The drawback of the traditional method relied on a lack of uncertainty consideration in the evaluation. Brief analysis, namely sensitivity analysis, is conducted which has been standard practice in the industry.

The main focus of the research, Real Options Analysis, was described in Chapter 3. After summarizing previous literature regarding real options analysis, we calculated the project value of our case study. The recent three approaches are explained and demonstrated in this chapter. Real options analysis was conducted to get the optimal decision of the monthly mine planning whether go mine, get more data by doing next exploration, or waiting for more information regarding the price change and cost prediction.

While in Chapter 4, we focused on the simulation method represented by the Multistage Stratified State Aggregation approach. The chapter started with modeling uncertainties deal with the mining industry in general. Furthermore, we detailed the economic, technical, and geological uncertainties in our case study. This methodology could be adapted for another mining location with adjustments based on the mineral, reserve property, mining method, and the corporate historical data regarding the cost of mining. It was developed to combine those three uncertainties and simulate them through java programming as a tool. Compared to the other two methods, this new approach's drawbacks and benefits are analyzed. The chapter analyzed the result of project value through traditional DCF and real options using the MSSA method. It contained a decision map and further analysis when the cost uncertainty was eliminated. This assumption was based on the ability of the company to control the expense in their project.

Chapter 5 covered the conclusion based on all analyses in our research.

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Chapter 2 Project Evaluation

2.1. Exploration

Project evaluation is a process of establishing the project's economic feasibility that requires a capital investment and making an investment decision. The industry whose revenue relied on ore selling is termed the resources industry. Ore is a metalliferous mineral, or an aggregate of metalliferous minerals, mixed with gangue which can be mined or processed at a profit (Hustrulid et al., 2013). Mineral and mining projects commonly experience unique evaluation hurdles. They are the difficulty of estimating reserves, catastrophic issues forecasting commodity prices and production costs, lengthy evaluation durations during production, unsure regulatory and environmental expenses, and, in most cases, the long project lives. Economic and technological circumstances can significantly alter the project value. The project's evaluation of technical, financial, social, environmental, and political components of the ecosystem must be determined before executing. They overlap to some level, but they form the economic foundation for an evaluation when taken together. In addition, every mine resource is unique; and tough to quantify the industry economics. Nevertheless, information is costly, especially in the exploration stage, which has no guarantee of success.

In general, the levels of feasibility progress in order from conceptual (pre-feasibility) to preliminary, intermediate, and final. To determine stages of feasibility, various firms employ different terminologies. Analysts determine whether to progress to the next stage and, ultimately, construct after each stage based on economics, environmental factors, and market timing. The first step of mining exploration is conceptual study, an initial assessment of a mining project. Flowsheet development, cost calculation, and production scheduling rely on limited test work and engineering design. Exploration data from drilling and sampling have to be enough to describe a resource accurately. This level of work is important for determining future engineering inputs and necessary investigations (Dyas, 2002).

A resource is a group of naturally existing materials in a concentrated form. It can be profitably extracted now or in the future. Geologic evidence is employed to determine the materials' location, grade, quality, and quantity. There are three types of resources which are measured, indicated, and inferred. Furthermore, the reserve is a fraction of a resource that meets minimum product specifications and may be reasonably expected to be profitably and technically produced at the time of determination. It is classified into two categories which are proven and probable (Hustrulid et al., 2013). As a result, the exploration stage, namely the feasibility study, has a critical role in transforming resources into reserves.

Predicting rock characteristics at unknown locations and anticipating the future flow behavior of complicated geological and engineering systems are the main problematic in resource exploration. Addressing this complex situation requires a variety of theories and assumptions (Pyrcz & Deutsch, 2014). A presumed orebody is a starting point for examination in exploration. Tonnage, grades, elevation, and physical address are used to define the potential deposit. Additionally, exploring a known orebody is carried out to discover new reserves and better understand current deposits to assure their dependability. Additional reserves almost always result in increased value straightforwardly (Runge, 1998). The last exploration step, feasibility study, provides technical, environmental, and commercial base for an investment decision. Iterative processes to optimize all critical elements of the project will be a procedure in feasibility study. The ultimate aim of the stage is identifying the production capacity, technology, investment and production costs, sales revenues, and return on investment.

Two standards of importance in the exploration can be defined for mines, i.e. (1) a minimum ore reserve equal to that required for all the years that the cash flows are projected in the feasibility report must be known with accuracy and confidence and (2) an ultimate tonnage potential, projected generously and optimistically, should be calculated to define the area adversely affected by the mining and within which dumps and plant buildings must not encroach (Hustrulid et al., 2013). Those expected exploration results can be generated after conducting drill hole surveys, sampling, splitting, assaying, logging drill holes, plotting, cross-sectioning, drill hole spacing, cutoff grade estimation, geostatistical analysis, and finally making reserve categories.

This research performed a project evaluation from exploration to economic decision recommendation. A case study was conducted at an underwater tin mine project in Indonesia operated by PT Timah, Tbk, as a mining company. Exploration of the company had been conducted since around 1970 and the available data was part of their concession. The exploration data existed at two locations, which later we termed as mid and south reserves. Exploration data consisted of drill collar and drill assay. The drill hole data amount was 55 holes with a total depth of 1459.6 meters. They were ranged from 4.8 meters under sea level (MUSL) to 47.3 MUSL. Additionally, tin content data were ranged from 0.008 kg/m^3 to 1.599 kg/m^3. The company have right to extract as well as abandon the resources. This research evaluated further the resource with its inherited uncertainties. The evaluation was started with modeling the resources using GEOVIA Surpac 6.8.1. After plotting the drill hole data, we composited it with its statistic (Figure 2.1) and variogram modeling (Figure 2.2) using the geostatistical methodology.

Statistics is the collection, organization, and interpretation of data and the conclusion and decision-making process. While geostatistics is a subfield of applied statistics. It encompasses the following concepts: spatial context (geological), spatial connections, volumetric support/scale, and uncertainty. The branch of knowledge was first developed by Krige (1951) and followed by Matheron (1963), which generated a statistical framework to resolve the relationship between vein width, the efficiency of exploration data in terms of sampling, measuring, and assaying. Geostatistics consists of three main concepts: regionalized variables, random function, and random variables. The regionalized variable is a function that takes a value at every point in the space of regionalization. However, the function varies irregularly from point to point in the space of regionalization. The random function is the set of random variables at all possible locations. The unique outcome that exists at every location realizes the random function. The grade may be considered a random variable that assumes a series of outcome values at unsampled locations. The series of possible outcome values for the random variable at each location is determined by a probability distribution (Nicholas, 2014).

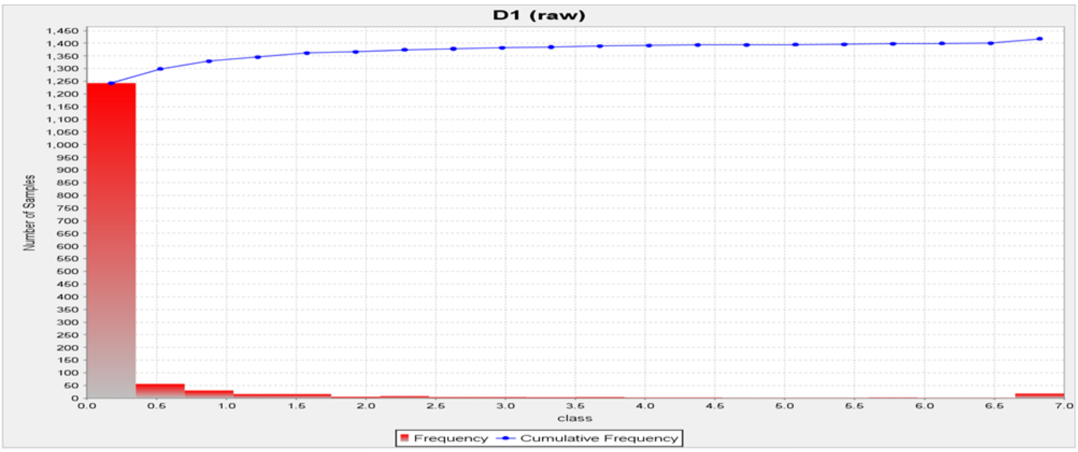


Figure 2.1 Basic Statistic Data

Variogram model or geostatistical semi variance is a function of difference over distance and is dissimilarity measurement. It is the half variance of grade differences for each drill hole pair with a certain direction mathematically expressed by Equation 2.1. The result of variance, $\gamma(h)$, will be more significant as the increase of distance (h). Nugget variance exists, which indicates the variance of the combined distribution of pairs of points separated by an infinitely small distance (h value almost 0). Typically, variogram experiments produced an asymptotic behavior termed sill. The distance when sill is reached is called range. This range is later used to define the recommended minimal distance of core drilling to get optimum data with a certain cost. Based on the model (Figure 2.2), the drill hole data has a range of influence of 30 m with sill 1.2 and nugget effect 0.3.

$$\gamma(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} [Z(x_i) - Z(x_{i+h})]^2 \qquad \text{Equation 2.1}$$

Where: n(h) = the number of couples separated by distance h,

h = distance between sample pairs,

$Z(x_i)$ = value of sample at x_i,

$Z(x_{i+h})$ = value of sample at x_{i+h}.

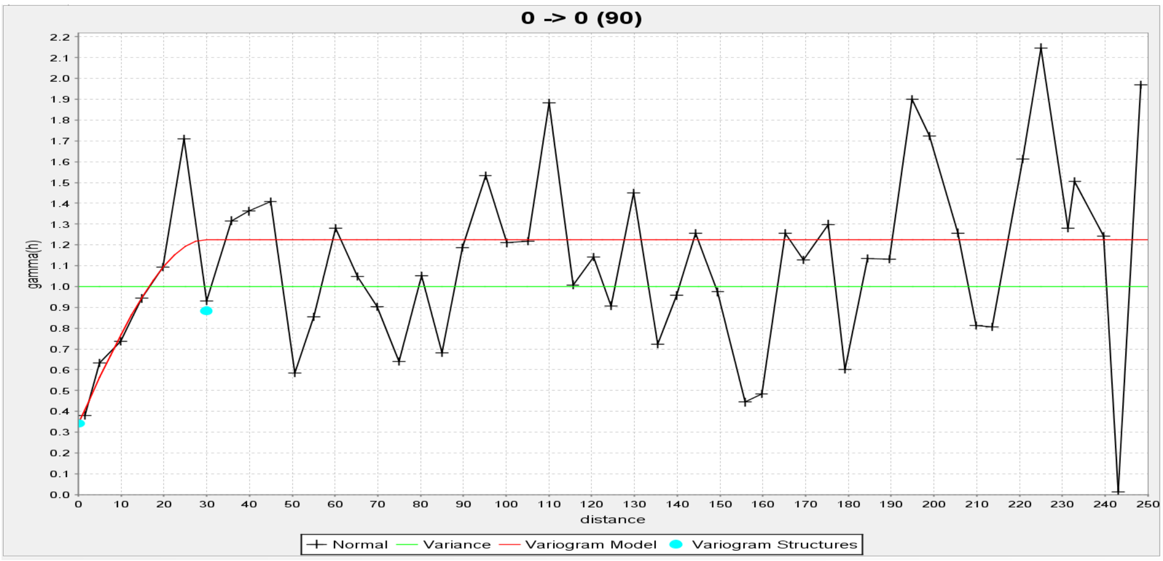


Figure 2.2 Variogram Model

Estimation is process of determining the best single value for a spatial attribute in an unsampled area. Kriging is a phrase coined by G. Matheron in 1963 in tribute to Danie Krige. It produces an interpolation function based on a covariance or variogram model derived from the data. Kriging method is defined as optimal regression of observed Z values of surrounding (real) data and weighted according to covariance values from semivariogram results. Local scale of kriging model is accurate and it takes local priority over global geographical variability. The value of estimation can be formulated as Equation 2.2 for simple kriging type.

$$Z^* = \sum_{i=1}^{n} \lambda_i Z_i \qquad \text{Equation 2.2}$$

Where Z* = Estimated block value

λ_i = unknown weight for value at i,

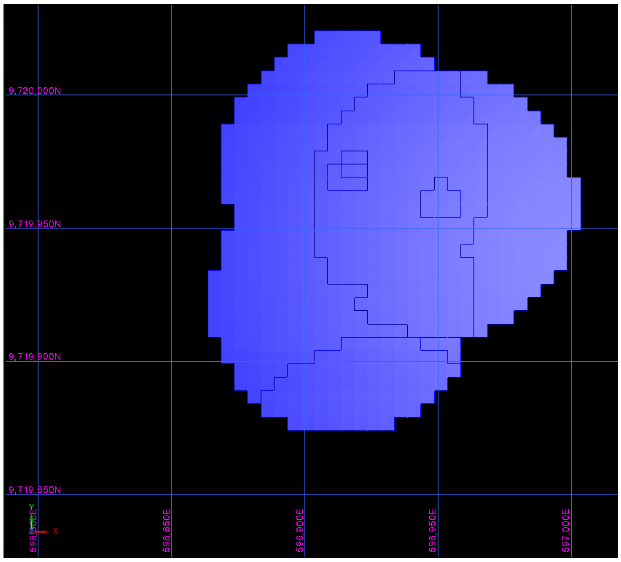
Z_i = measured value at i.

By contrast, simulation is the process of achieving one or more acceptable values for a reservoir attribute at an unsampled location. The idea of conditional simulations is to build a representation of the phenomenon that is consistent with the data observed sampling interval, as kriging is, and yet reproduces the local fluctuations. The procedure of globally accurate and consistent with global data are used. Conditional simulation generates value at sampled points and produces the same dispersion characteristics of the original data set or mean, variance, and covariance or semivariogram. The conditional simulation generates sets of realization that each reproduces histogram, spatial variability, and known data of a variable. Each of them is independent and equal to be drawn from referred set. The obtained block model will be evaluated and compared with the results.

Inputting parameters of compositing data and variogram model resulted kriging model (Figure 2.3, and 2.4) and conditional simulation model (Figure 2.5 and 2.6). The resource estimation through Kriging and Conditional simulation method is described at Appendix A and summarized at Table 2.1.

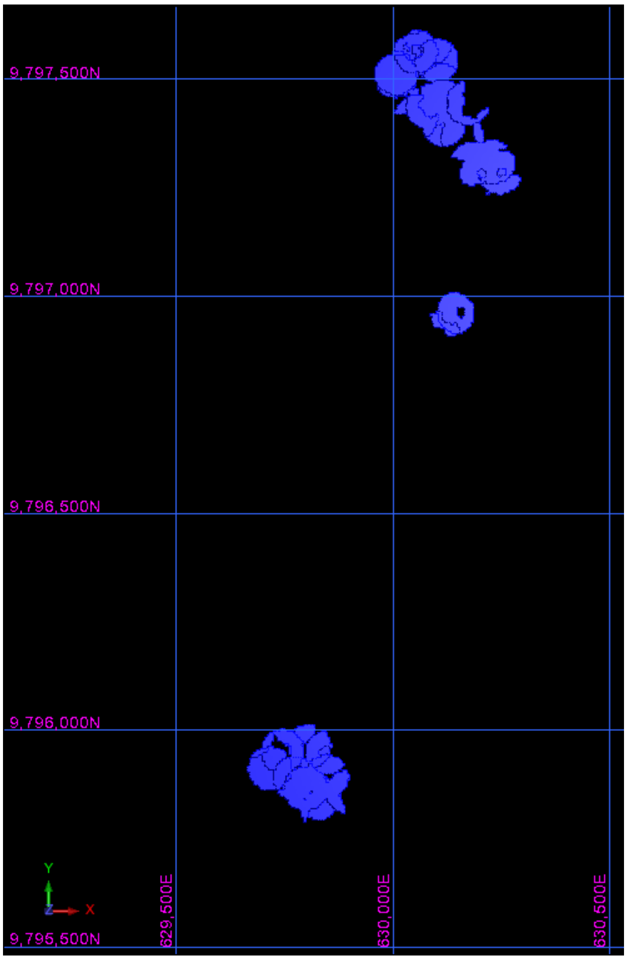
Table 2.1 Resource Summary

Location	Method	Tonnage	Grade (%)
Mid	Kriging	4,824,000	0.013
Mid	Conditional Simulation	11,503,500	0.016
South	Kriging	728,250	0.010
South	Conditional Simulation	1,205,250	0.019



Colour	Attribute values
r=0.00 g=0.00 b=1.00	0.00 -> 0.20
r=0.00 g=0.50 b=1.00	0.20 -> 0.40
r=0.00 g=1.00 b=1.00	0.40 -> 0.60
r=0.00 g=1.00 b=0.50	0.60 -> 0.80
r=0.00 g=1.00 b=0.00	0.80 -> 1.00
r=0.50 g=1.00 b=0.00	1.00 -> 1.20
r=1.00 g=1.00 b=0.00	1.20 -> 1.40
r=1.00 g=0.50 b=0.00	1.40 -> 1.60
r=1.00 g=0.00 b=0.00	1.60 -> 1.80

Figure 2.3 Ordinary Kriging Model for South Reserve



Colour	Attribute values
r=0.00 g=0.00 b=1.00	0.00 -> 0.50
r=0.00 g=0.29 b=1.00	0.50 -> 1.00
r=0.00 g=0.57 b=1.00	1.00 -> 1.50
r=0.00 g=0.86 b=1.00	1.50 -> 2.00
r=0.00 g=1.00 b=0.86	2.00 -> 2.50
r=0.00 g=1.00 b=0.57	2.50 -> 3.00
r=0.00 g=1.00 b=0.29	3.00 -> 3.50
r=0.00 g=1.00 b=0.00	3.50 -> 4.00
r=0.29 g=1.00 b=0.00	4.00 -> 4.50
r=0.57 g=1.00 b=0.00	4.50 -> 5.00
r=0.86 g=1.00 b=0.00	5.00 -> 5.50
r=1.00 g=0.86 b=0.00	5.50 -> 6.00
r=1.00 g=0.57 b=0.00	6.00 -> 6.50
r=1.00 g=0.29 b=0.00	6.50 -> 7.00
r=1.00 g=0.00 b=0.00	7.00 -> 7.50

Figure 2.4 Ordinary Kriging Model for Mid Reserve

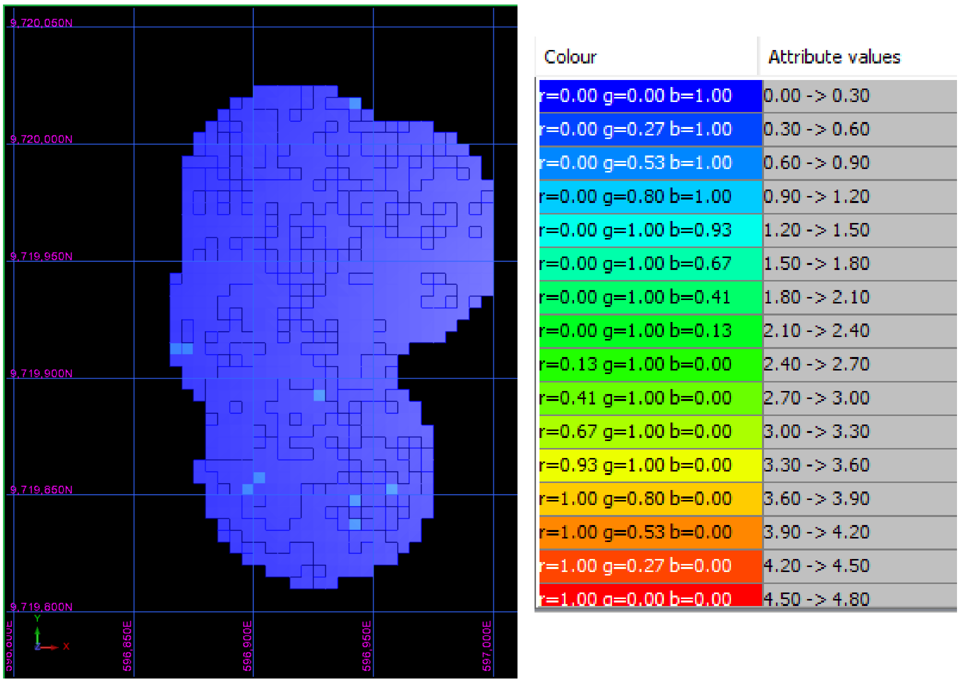


Figure 2.5 Conditional Simulation Model for South Reserve

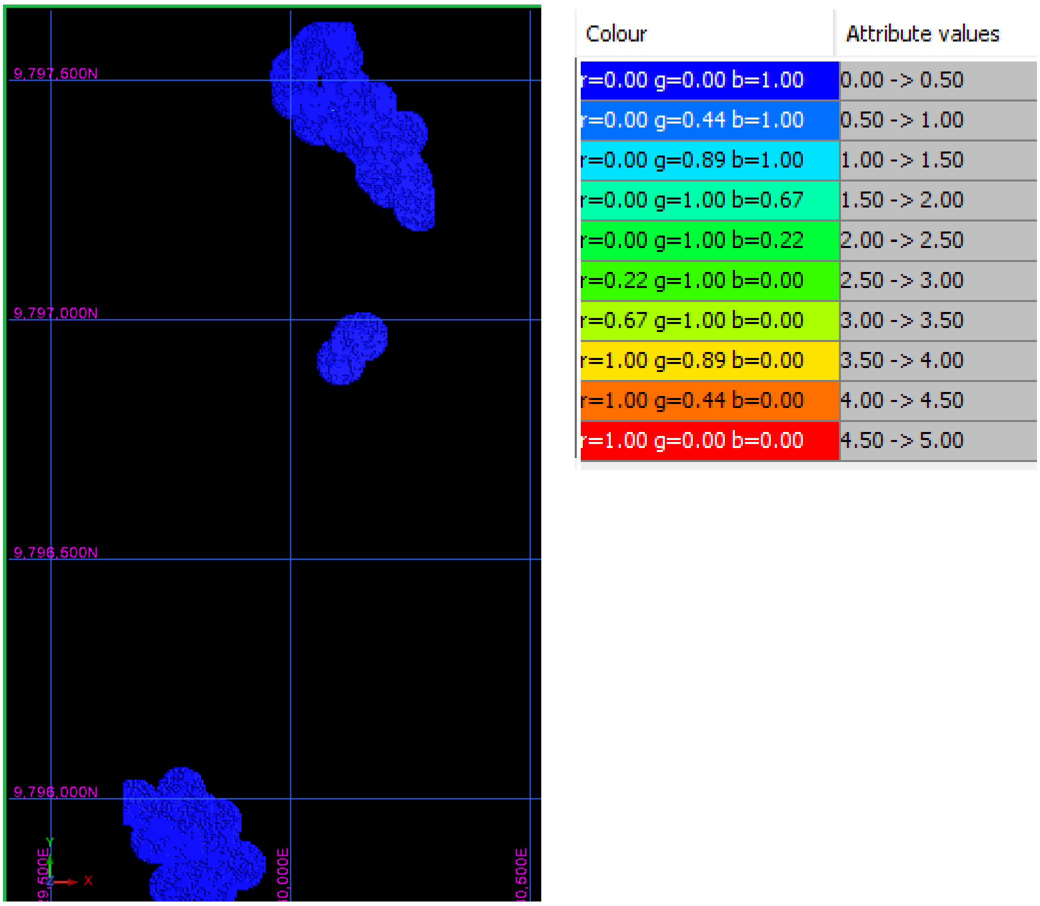


Figure 2.6 Conditional Simulation Model for Mid Reserve

2.2. Mine Planning

Mine planning is defined as a systematic process set out in a three-dimensional flowchart in Figure 2.7. The process consists of a similar series of steps in each phase, each undertaken in the same order. Three such phases are illustrated in Figure 2.7. However, in practice, there may be any number of phases. The same series of steps are undertaken in varying amounts of detail, depending on the precision, economic action, or decision being sought (Runge, 1998). The ultimate aim of mine planning is to get the value of the project, which is feasible to be extracted practically. Once the reserve geology and quality are well understood, then mine planning can begin concerning the following: (1) orientation of mine works; (2) access to the reserve; (3) determination of opening widths; (4) selection of mine method; (5) selection of development and secondary mining heights; (6) appropriate inter-burden thicknesses; and (7) examining the stability of the mine (Newman et al., 2020).

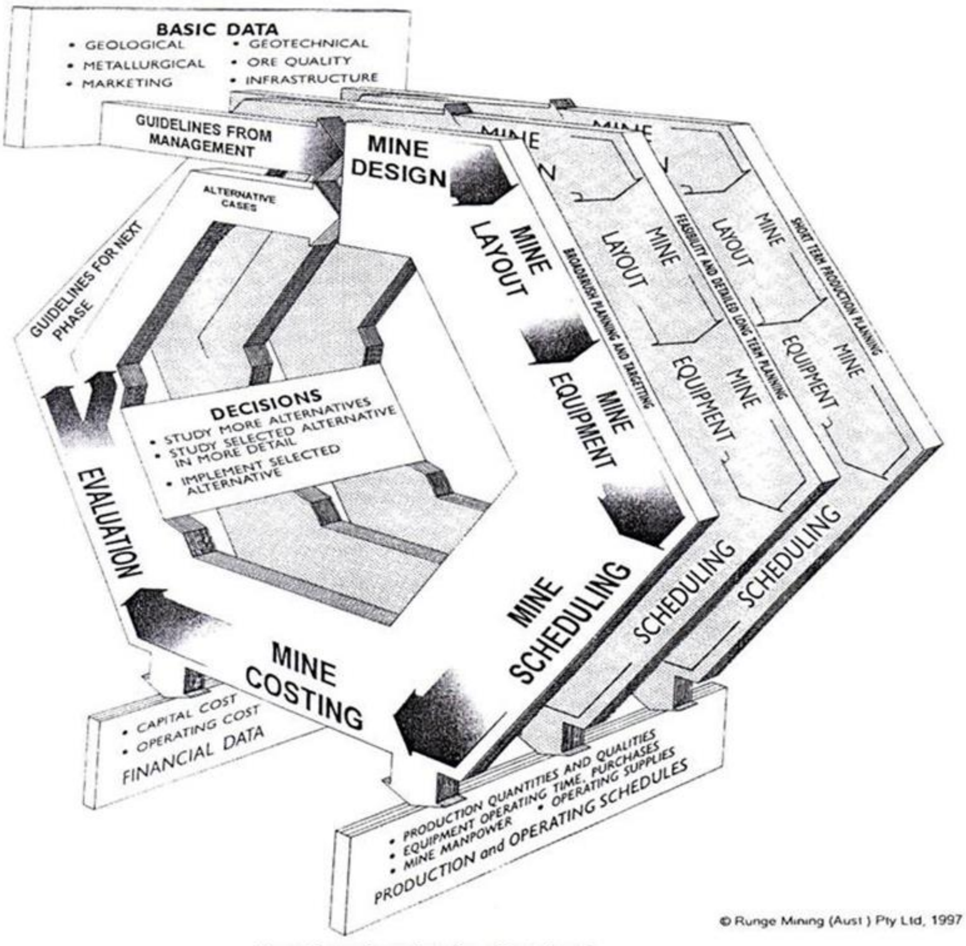


Figure 2.7 Iterative Mine Planning Flow Chart

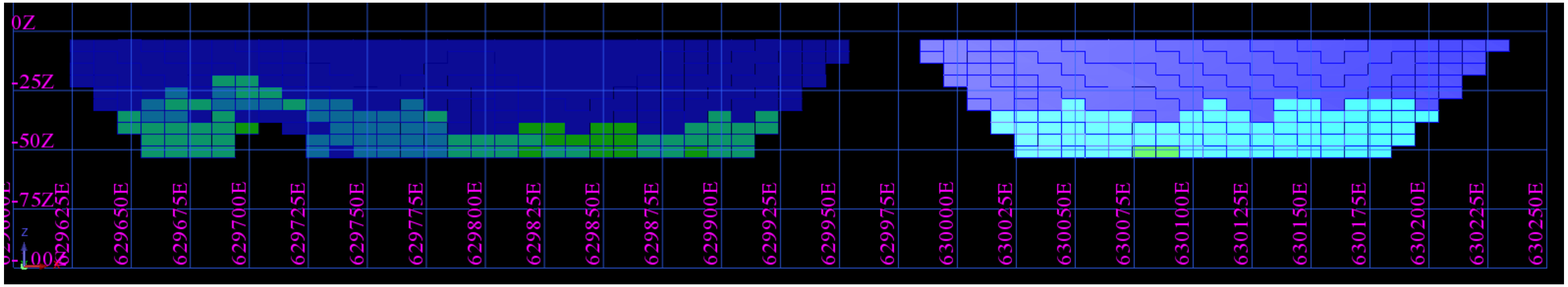
In this research, monthly mine planning is conducted for underwater mining with dredge as main equipment. We developed a mine planning simulation using GEOVIA Surpac 6.8.1 and utilized GEOVIA Whittle 4.6 mining software to optimize the mine design. Optimizer software can reduce the iterative activity of mine planning. It employed Lerch - Grossman algorithm to get the maximum NPV. The algorithm pit limit based on fixed slope angles governed by block dimensions. The parameter we used to develop the mine plan is described in Table 2.2. The cost parameter is obtained from averaging historical cost with similar mining method and equipment. We simulated for two reserve locations using the Kriging model or Conditional Simulation model (Table 2.3 and Appendix B). The design result of mine planning is shown at Figure 2.8 and Figure 2.9.

Table 2.2 Mine Planning Parameters

Parameters	**Unit**	**Value**
Natural Slope	Degree	45
Tin Selling Price	USD/Ton	23,199
Mining Operation Cost	USD/ Ton Ore	1.46
Mining Recovery		0.9
Mining Dilution		1.1
Processing Cost	USD/kg/m3	1.15
Processing Recovery		0.9
Selling Cost	USD/kg/m3	0.5

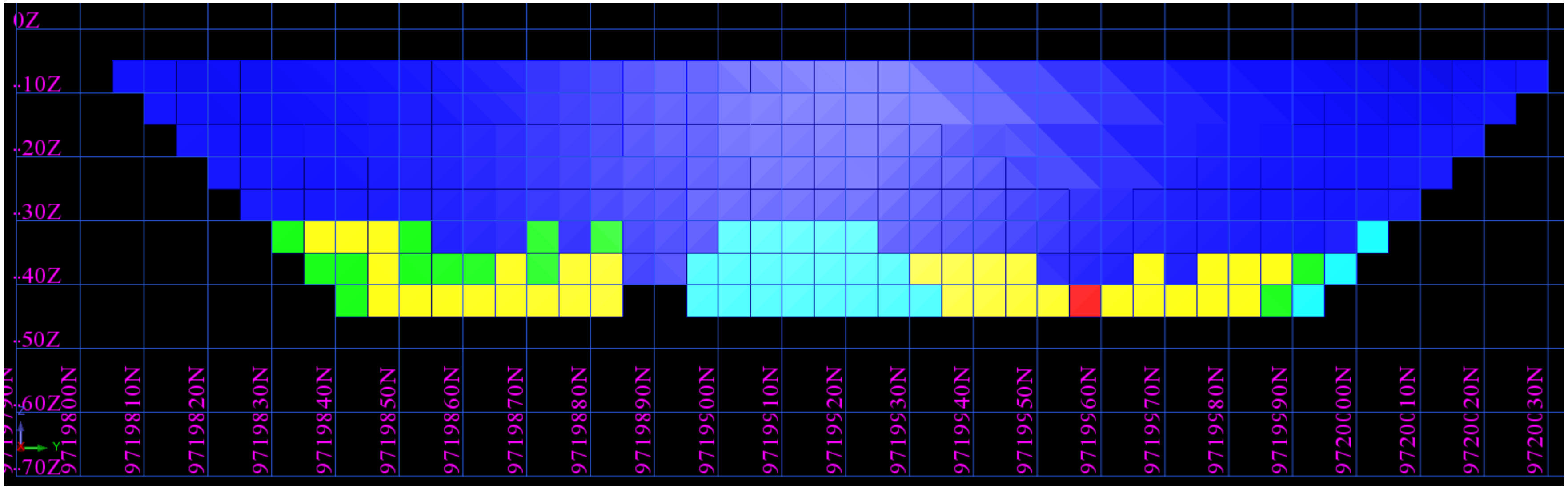
Table 2.3 Ore and Waste Mine Production

Location	Volume	Tones	Grade (Kg/ M^3)	Tin (Ton)	NPV (USD)
Ordinary Kriging Model					
mid	748,000	2,244,000	0.36	269.28	1,147,183.00
south	172,000	516,000	0.17	28.896	-279,926.00
Conditional Simulation Model					
mid	3,680,500	11,041,500	0.39	1,428.03	7,294,826.00
south	580,875	1,742,625	0.36	208.53	882,590.00



Colour	Attribute values
r=0.00 g=0.00 b=1.00	0.00 -> 0.50
r=0.00 g=0.67 b=1.00	0.50 -> 1.00
r=0.00 g=1.00 b=0.67	1.00 -> 1.50
r=0.00 g=1.00 b=0.00	1.50 -> 2.00
r=0.67 g=1.00 b=0.00	2.00 -> 2.50
r=1.00 g=0.67 b=0.00	2.50 -> 3.00
r=1.00 g=0.00 b=0.00	3.00 -> 3.50

Figure 2.8 Ultimate Pit Limit Design for Mid Reserve



Colour	Attribute values
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r=0.00 g=1.00 b=1.00	0.50 -> 1.00
r=0.00 g=1.00 b=0.00	1.00 -> 1.50
r=1.00 g=1.00 b=0.00	1.50 -> 2.00
r=1.00 g=0.00 b=0.00	2.00 -> 2.50

Figure 2.9 Ultimate Pit Limit Design for South Reserve

2.3. Mining Economics

The Discounted Cash Flow (DCF) approach is one of the most frequent methods for evaluating a mining project. For most mining prospects, DCF methodologies are used to make investment decisions (Fig. 2.10). This strategy has been a popular tool for numerous decades for completing appraisals and allocating scarce funds. It is based on cash flow and is simple to comprehend. The DCF method assesses the entire project by modifying or discounting the project net cash flow to account for risk and time. The bigger the investment risk, the higher the discount rate. DCF approaches do not allow for management flexibility and it assures parameters throughout the project. Decisions must be taken "now or never," and the use of an appropriate discount rate is critical.

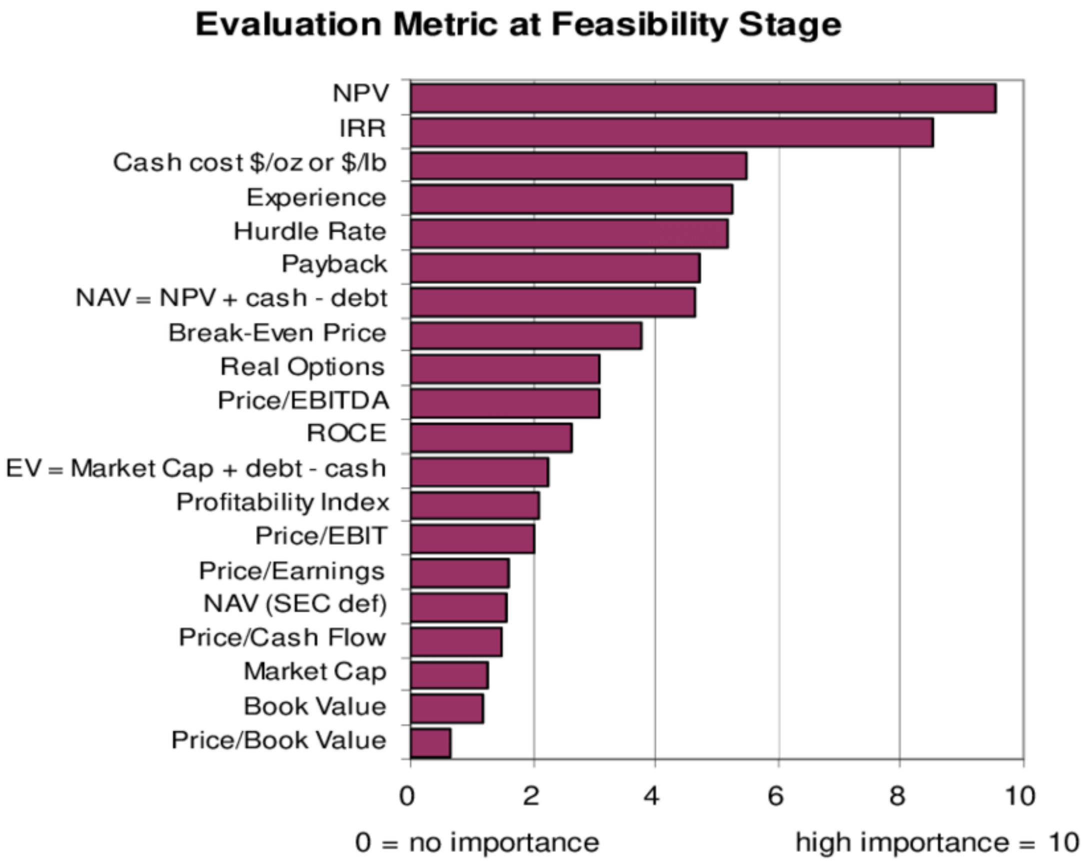

Figure 2.10 Evaluation Metric at Feasibility Stage

Source: Whittle D., et al., 2005

Evaluating a project is a multidisciplinary task and is not solely the domain of any profession or management. It involves engineering economics, capital budgeting, financial management, and strategic planning (Crundwell, 2008). The key concept of the resource industry is extraction leading to a profit. Therefore, several parameters in the economic modeling of mining projects: capital expenditure, operating cost, and commodity price have to be defined. The capital expenditure is the sum of money required to develop and install a manufacturing facility. At the same time, operating costs are those costs that are incurred in the direct manufacture of the items. The commodity price is from the world trade price represented in London Metal Exchange. The cash flow model summarizes the money stream in and out of a planned project. While the cost is resumed at Appendix D.

Well-known economic theory, the time value of money, explained that the current value of money is bigger than the future and less than the past (Equation 2.3). There are three reasons for this: inflation, risk, and liquidity (Crundwell, 2008). The Net Present Value (NPV), as expressed in Equation 2.4. is the total of all cash flows discounted at present using the time value of the money. The investor's value is increased when the net present value is positive. If it is less than zero, the investor's interest is expected to be lost.

$$\mathrm{FV} = \mathrm{PV}(1+r)^t \qquad \text{Equation 2.3}$$

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t} - I \qquad \text{Equation 2.4}$$

Where FV = future value

PV = present value

r = discount rate

t = number of period

NPV = Net Present Value

CF = cash flow

I = initial investment

The evaluation of cash flow in the traditional DCF method is simply the subtractions revenues and costs. The revenues stream only comes from selling material with a certain selling price. Therefore, revenue is governed by grade, throughput, recovery, and metal or product price. Of these, price is: (a) by far the most difficult to estimate and (b) the one quantity largely outside the estimator's control. Even ignoring inflation, selling prices are widely variable with time. While for the costs can be divided into two big groups: capital investment and operating cost. Generally, accuracy in capital or operating cost estimating goes back to accuracy in quantities, reliable unit prices, and adequate provision for indirect or overhead items. Based on the evolution of commodity prices, the mine operation can be altered (Rimélé et al., 2020).

Traditional DCF techniques do not account for decisional flexibility in project capital planning. DCF analysis fails to account for the value of genuine alternatives inherent in capital budgeting, which explains why it does not consider the true worth of an investment (Trigeorgis, 1993). Misuses of DCF methods include discounting real flows with nominal interest rates, failing to implement inflation correctly; applying unnecessary risk-adjustment factors, failing

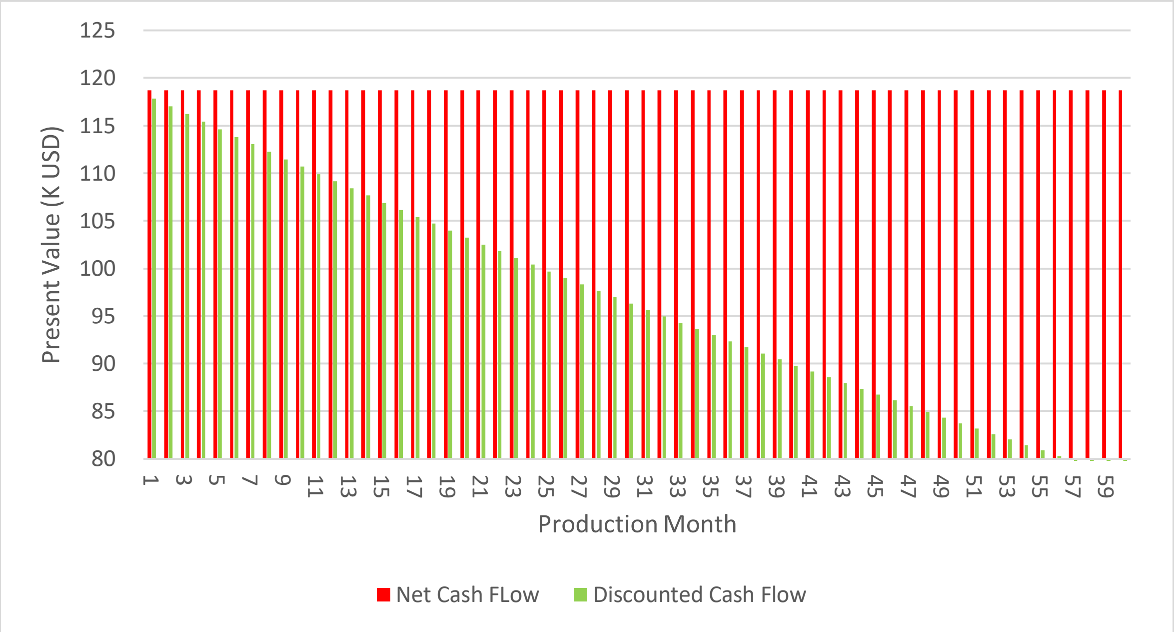

Figure 2.11 Mid Monthly Cash Flow Model

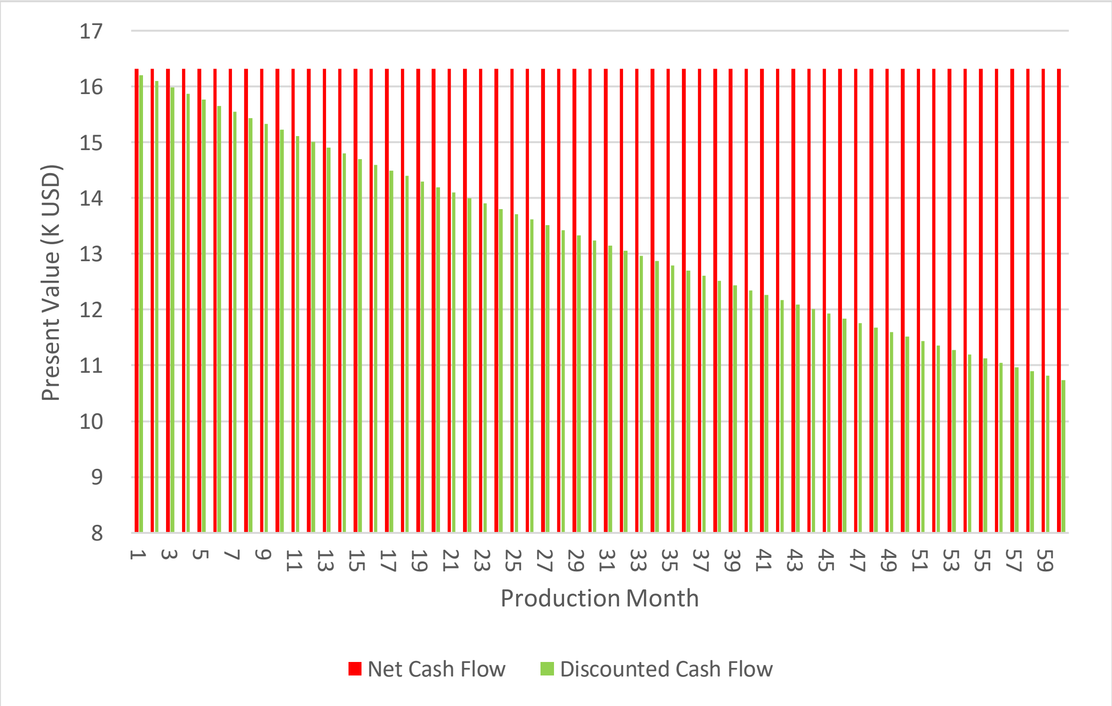

Figure 2.12 South Monthly Cash Flow Model

to recognize that risk diversification should not be considered; using unsuitable criteria in measuring profitability, such as internal rate of return, which frequently yield erroneous conclusions; and frequently gauging cash flows using standardized accounting principles.

In the case study, the evaluation was conducted in February 2021. Therefore, the used price was flat on January 2021. While for the operating cost was resumed from the company with a similar operation method and similar location. The overall operation did not cause initial capital expenditure. The research evaluated the reserve that had been explored before. The company had the right to execute or abandon the project. As we assumed the production was flat in 5 years of operation, the cash flow and cash flow models are shown in Table 2.4, Table 2.5, Figure 2.11, and Figure 2.12. It resulted in Net Present Value of 6.6 MUSD and 0.9 MUSD for mid and south reserve, respectively.

Table 2.4 Cash Flow Model for Mid Reserve

Component	Unit	Unit value	Total value	1	2	3	…	58	59	60
Production Ore	1000 ton		7,204	120	120	120	…	120	120	120
Grade	kg/m3		0.594	0.594	0.594	0.594	…	0.594	0.594	0.594
Tin Got	ton		1,426	23.77	23.77	23.77	…	23.77	23.77	23.77
Selling Price	usd/ton		23,199	23,199	23,199	23,199	…	23,199	23,199	23,199
Gross Revenue	1000 usd			551.51	551.51	551.51	…	551.51	551.51	551.51
Mining Cost			12,785	213.08	213.08	213.08	…	213.08	213.08	213.08
Processing Cost			6,738	112.30	112.30	112.30	…	112.30	112.30	112.30
Operating Cost	1000 usd		19,523	325.38	325.38	325.38	…	325.38	325.38	325.38
Net Income Before Tax	1000 usd			226.13	226.13	226.13	…	226.13	226.13	226.13
Tax	1000 usd	28%		63.32	63.32	63.32	…	63.32	63.32	63.32
Cash Flow	1000 usd			118.69	118.69	118.69	…	118.69	118.69	118.69
Discounted value	1000 usd	0.25%		$118.40	$118.10	$117.81	…	$102.69	$102.43	$102.18

Net Present Value = 6.605 Million USD

Table 2.5 Cash Flow Model for South Reserve

Component	Unit	Unit value	Total value	1	2	3	…	58	59	60
Production Ore	1000 ton		1,123	18.72	18.72	18.72	…	18.72	18.72	18.72
Grade	kg/m3		0.558	0.558	0.558	0.558	…	0.558	0.558	0.558
Tin Got	ton		208.88	3.48	3.48	3.48	…	3.48	3.48	3.48
Selling Price	usd/ton		23,199	23,199	23,199	23,199	…	23,199	23,199	23,199
Gross Revenue	1000 usd			80.76	80.76	80.76	…	80.76	80.76	80.76
Mining Cost		1.77	1,993.08	33.22	33.22	33.22	…	33.22	33.22	33.22
Processing Cost		0.87	986.80	16.45	16.45	16.45	…	16.45	16.45	16.45
Operating Cost	1000 usd	3.35	2,979.87	49.66	49.66	49.66	…	49.66	49.66	49.66
Net Income Before Tax	1000 usd			31.10	31.10	31.10	…	31.10	31.10	31.10
Tax	1000 usd	28%		8.71	8.71	8.71	…	8.71	8.71	8.71
Cash Flow	1000 usd			16.32	16.32	16.32	…	16.32	16.32	16.32
Discounted value	1000 usd	0.25%		$16.28	$16.24	$16.20	…	$14.12	$14.09	$14.05

Net Present Value = 0.908 Million USD

2.4. Sensitivity Analysis

The risks related to mining are diverse and complicated, with the orebody constituting the primary source of risk. Mining is distinct from most other industries in that product knowledge relies primarily on estimations that carry a degree of uncertainty. World commodity prices heavily influence possible income fluctuations and, hence, the amount of the commercial mineral inventory (Dominy, 2016).

Sensitivity analysis is the process of changing one or more elements to examine how the variation affects the project's value. While sensitivity analysis helps to understand the implications of uncertainty, it does not provide a project value adjusted for the uncertainty. One of the most important benefits of sensitivity analysis is that it identifies the elements that have the greatest impact on the economics of a project. The analysis enables assessors to collect more information more effectively.

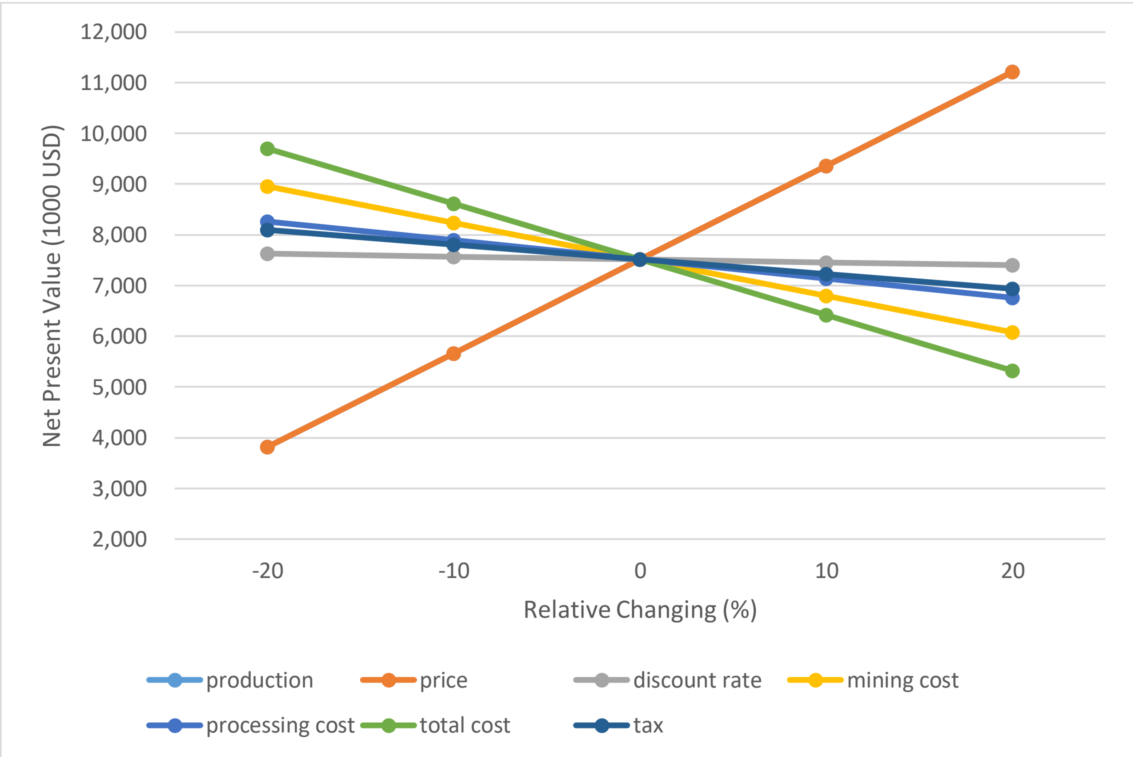

Figure 2.13 Sensitivity Analysis

We conducted a sensitivity analysis on price, reserve, and cost variables on our case study (Figure 2.13). Price and production were the most sensitive parameters, followed by total cost, mining cost, processing cost, tax, and discount rate. However, because sensitivity analyses are usually conducted on a single ‘best estimate’ of the resource model, it should reflect that level of local inaccuracy and could be misleading decision-making. Therefore, combining more than one uncertainties evaluation is necessary and analysis based on simulation should be conducted to anticipate an unpredictable situation in the future.

Sensitivity analysis can understand project risk straight forward. In addition, all stochastic variables need to be considered in a single evaluation model with all possible permutations (Nicholas, 2014). Practically, general mining companies conduct sensitivity analysis to measure the effect of parameter changing. It runs analyses that reflect various commodity prices, reserve amount, and operating and capital costs to determine the effect of such variations. In very simplistic terms, sensitivity analysis is 'what if' analysis, which is an important notion at the core of any application of decision tools and may be applied to a wide range of uses. One-way sensitivity analysis is the simplest form of sensitivity analysis where a given amount varies one value in the model to examine the impact of the change on the model's results (Taylor, 2009).

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Chapter 3 Real Option Analysis

3.1. Real Option Analysis Framework

An option is a financial derivative whose value depends on the price of a stock. Option holders have rights but no obligation to execute their option in the maturity date. These alternatives are quantified by option calculation to justify option value. The adaptation of option calculation in the actual project is called real options (RO). For instance, if a company decides to invest in a mine, they exercise right. They lose the right, but they earn the asset. Discounted Cash Flow (DCF) method theoretically assumed static decision over project life time without ability to change decision. In contrast, RO assumed dynamic future decisions when the uncertainties resolved. That managerial flexibility is not accommodated in the traditional DCF methodology. For instance, the operation of a project can be contracted, delayed, or abandoned and real options discount uncertain cash flows at the correct rate (Crundwell, 2008).

The analysis of options was introduced by Fisher Black and Myron Scholes to analyze a derivative market product (Black and Scholes, 1973). It had been popular with the lattice method, which was developed further. The real options theory was largely accepted in the financial literature, and many scholars sought to alter it (Table 4.1). Cox, Ross, and Rubinstein developed a basic economic concept of option pricing under the non-arbitrage assumption in 1979 and a simple and efficient numerical approach for valuing options (Cox et al., 1979). Myers (1977) stated real option first time in the study of corporate debt policies, which account for the flexibility of the company management. Mun (2006) added that a real option is a systematic approach and integrated solution using financial theory, economic analysis, management, decision science, statistic, and econometrics to evaluate real asset dynamic and uncertain.

Brennan and Schwartz (1985) used an option pricing model to analyze a natural resource project for the first time. Natural resources, they said, contain much unpredictability in terms of resources and cost. According to the Brennan and Schwartz model, continuous-time non-arbitrage methods and stochastic control theory may be used to assess such enterprises and find the best strategies for creating, managing, and abandoning them. Trigeorgis offered examples of applied actual options valuation (Trigeorgis, 1993). They demonstrated how to postpone, enlarge, and shut a variety of investment project decisions, particularly in the natural resource sector. They indicated that commodity price changes have a major impact on natural resource investment decisions. All prior empirical research on ROV and DCF in mining operations is included in Table 3.1. This thesis employed ROV in a mining project with a combination of uncertainties which will be the originality of the research.

Table 3.1 Empirical Example of the ROV and DCF for Commodities

No	Year	Author (s)	Method	Commodity	Mine/project name	Project location
1	1985	M J Brennan, E S Schwartz	ROV	Copper	Hypothetical	Not available
2	1985	M J Brennan, E S Schwartz	ROV	Gold	Hypothetical	Not available
3	1986	S K Palm, N D Pearson	DCF.ROV	Copper	Not available	Not available
4	1988	J L Paddock, D R Siegel, F L Smith	DCF.ROV	Oil	Gulf of Mexico	USA
5	1992	N Kulatilaka, A J Marcus	DCF.ROV	Oil, gas	Not available	Not available
6	1992	B Cavender	DCF.ROV	Gold	Hypothetical	USA
7	1993	D G Laughton, H D Jacoby	DCF.ROV	Oil	Not available	Not available
8	1993	J L Mardones	DCF.ROV	Copper	Not available	Chile
9	1993	E Pickles, J L Smith	DCF.ROV	Oil	Not available	USA
10	1994	N Kulatilaka, L Trigeorgis	DCF.ROV	Oil	Hypothetical	Not available
11	1996	D G Laughton	DCF.ROV	Copper	Not available	Not available
12	1996	M Samis, R Poulin	DCF.ROV	Gold	Not available	Not available
13	1997	S Frimpong J Whiting	DCF.ROV	Copper	Confidential	Not available
14	1998	G Salahor	DCF.ROV	Gas	Not available	Not available
15	1998	M Samis, R Poulin	DCF.ROV	Copper	Not available	Not available
16	1998	G Cortazar, J Whiting	DCF.ROV	Copper	Not available	Not available
17	1998	W S Dunbar, S Dessureault	ROV	Underground mine	Not available	Not available
18	1998	W S Dunbar, S D, M Scoble	ROV	Gold	Not available	Not available
19	1998	F S Sagi. EE Hiob. S Jones	ROV	Copper	Not available	Not available
20	1998	Skelly FE Smith, K F McCardle	DCF.ROV	Gold	Lihir Island	Papua New Guinea
21	1999	J E Smith, K F McCarkle	DCF.ROV	Oil, gas	Hypothetical	Not available
22	1999	F P Camus, C W Pelley	ROV	Copper	Not available	Not available
23	2000	R T McKnight	DCF.ROV	Copper	Not available	Not available
24	2001	M E Slade	DCF.ROV	Copper	21 mines	Canada
25	2001	S Faiz	DCF.ROV	Oil	Chevron Texaco	USA
26	2002	J A Drieza, J Kicki, P Saluga	DCF.ROV	Zinc, lead	Olkusz Pomorzany	Poland
27	2002	A Moel, P Tufano	ROV	Gold	285 mines	North America
28	2003	D Colwell, T Henker, J Ho, K Fong	DCF.ROV	Gold	27 companies	Australia
29	2004	W bailey, A B, S Faiz, Srinivasan, H Weeks	ROV	Gas	Elba Island	Georgia
30	2004	S Kelly	DCF.ROV	Gold	41 mines	Australia

31	2005	V Blais, R Poulin, M Samis	DCF.ROV	Copper, gold	Not available	Canada
32	2006	M Samis, G A Davis, D Laughton	DCF.ROV	Copper	Not available	Not available
33	2007	J Hall, S Nicholls	DCF.ROV	Coal	Hypothetical	Not available
34	2007	S Dessureault, V N Kazakidis, Z Mayer	DCF.ROV	Nickel, copper	Sudbury, Western Arizona	Canada, USA
35	2007	P Guj, R Garzon	DCF.ROV	Nickle	Not available	Not available
36	2007	G Dogbe, S Frimpong, J Szymanski	DCF.ROV	Copper	Hypothetical	Not available
37	2008	S Shafiee, E Topal	DCF.ROV	Gold	Hypothetical	Australia
38	2009	Li Shu-xing, Knights Peter	DCF ROV	Not available	Hypothetical	Not available
39	2009	S.A. Abdel Sabour and G. Wood	DCF ROV	Copper, Gold	Not available	Canada
40	2009	Akbari Afshin Dehkharghani, Osanloo Morteza, Shirazi Mohsen Akbarpour	DCF ROV	Copper	Not available	Not available
41	2011	S. A. Abdel Sabour and R. Dimitrakopoulos	DCF ROV	Copper	Not available	Canada
42	2014	Hesam Dehghani, Majid Ataee-pour, Akbar Esfahanipour	DCF ROV	Gold, Copper	Grasberg	Indonesia
43	2015	Snehamoy Chatterjee, Manas Ranjan Sethi, Mohammad Waqar Ali Asad	DCF ROV	Iron	Not available	India
44	2015	Ajak Duany Ajak, Erkan Topal	ROV	Iron	Not available	Australia
45	2017	Jyrki Savolainen	ROV	Not available	170 mines	Not available
46	2017	Oscar Miranda, Luiz E. Brandao, Juan Lazo Lazo	DCF ROV	Silver	Not available	Peru
47	2018	Xiaoran Liu and Ehud I. Ronn	ROV	Renewable Energy	Not available	China
48	2019	Aldin Ardian and Mustafa Kumral	DCF ROV	Gold	Not available	Canada, Indonesia
49	2019	Shiwei Yu, Zhenxi Li, Yi-Ming Wei, Lancui Liu	DCF ROV	Geothermal	Xiong New Area	China
50	2020	Giorgio Locatelli, Mauro Mancini, Giovanni Lotti	DCF ROV	Energy	Not available	Not available

Result of a project or underlying asset will remain uncertain until the end of the project period. Therefore, the owner of the right does not know whether their choice will be profitable at the option's maturity. The strike cost for real option is making the production facility to start the project. Real option valuation parameter is resumed at Table 3.2. Option value can be measured in various ways, such as path-dependent simulation, closed-form formulas, partial differential equations, and lattice methods. Real option concept based on the manager's flexibility to take action when information is obtained. The method implies that the manager should defer or abandon the mine project when the project will not profitable respecting uncertainty model.

The option evaluation in the mining project is divided into each mining stage. It contained exploration, development, extraction, and abandonment. For instance, delay option type is put option with the time maturity from acquisition to closing. The other option type is the call option with the mature period, which is acquisition to the next stages. In the extraction stage, the partition came to mine design, mine planning, temporary close, expansion, defer building, and switch option. Every type of options analysis has its particular framework, as calculating option value is various. In this thesis, the calculation of option is focus on the wait to operate option which the manager has right to delay operation considering uncertainty of the project.

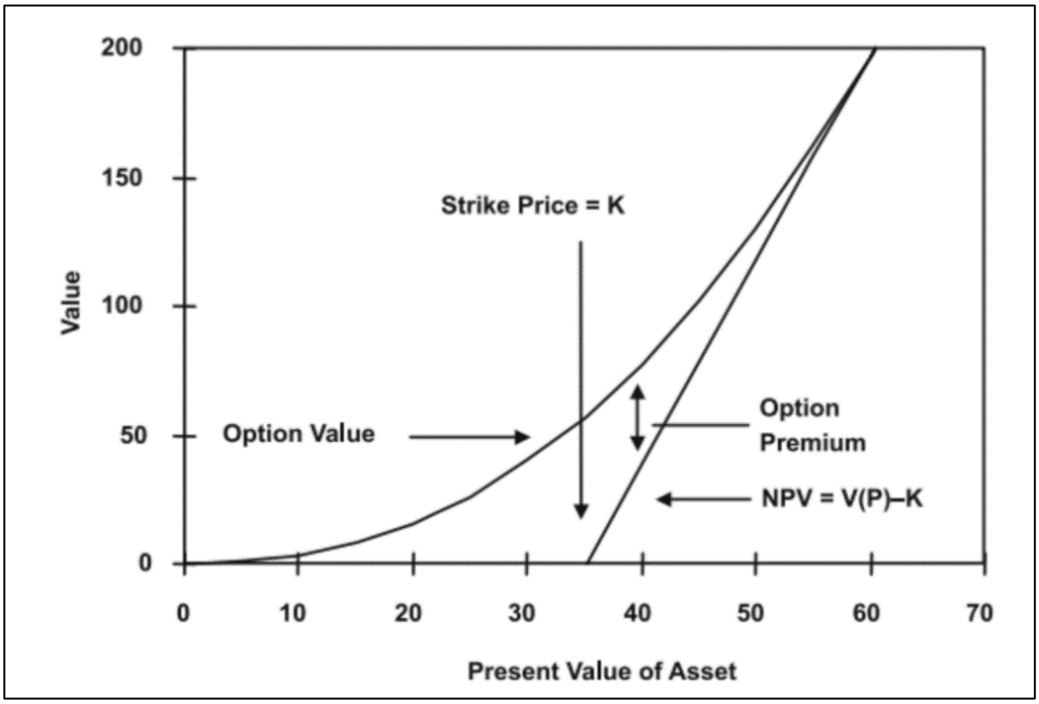

Figure 3.1 Result of option analysis

(Crundwell, 2008)

Table 3.2 Option Parameters

Financial Option Parameter	Real Option Parameter	Definition
Strike Price	Capital Cost	The price to get the option
Maturity	Project period	Length of the option
Underlying Asset	Present Value	The asset value
Call Option	Invest option	Right to buy an asset
Put Option	Abandon option	Right to sell an asset
Long Position	Investor	Owner of an option
Short Position	-	The writer who buy the option
European Option	-	The option can only be exercised at maturity
American Option	American Option	The option can be exercised at any time before the option expires

Cortazar has provided a case study of real options in mining exploration which accounted for geological and economical uncertainties (Cortazar et al., 2001). In line with this research, there were three sources of uncertainties: geological, technical and price uncertainties. Recently, the evidence of uncertainty in mining project valuation has been highlighted. Their sources are recovery, grade, commodity price, discount rate, and mining cost. They drive the successes of mining projects when the company is accurate in predicting. However, simulating multiple uncertainty parameters can overcome the difficulty of prediction (Ardian & Kumral, 2020). Theoretically, the ultimate result of the option value can be described in Figure 3.1. It consist of option value, strike price, Net Present Value and the option premium. The asset value will be varied as uncertainty parameter is changed.

Inputs of cash flow are sources of uncertainty. Technical parameters refer to production uncertainty which consists of the reserve, mining, and metallurgical uncertainty. While selling price, capital cost, and variable cost lead to economic uncertainty. Real option foundation relies on the flexibility of management to take action when uncertainty is resolved in the future time.

3.2. Black Scholes (BS) Model

Black and Scholes model employed the capital asset pricing model to derive a link between the market's necessary return on the option and the required return on the stock. Black Scholes method is based on the potential underlying asset, and the cost expensed during the project (Hull, 2015).

The assumption of the approach is:

1. The stock price follows the Geometric Brownian Model process with growth (μ) and volatility (σ) constant (Equation 3.1),
2. The short-selling of securities with full use of proceeds is permitted,
3. There are no transaction costs or taxes. All securities are perfectly divisible,
4. No dividends during the life of the product,
5. There are no riskless arbitrage opportunities,
6. Security trading is continuous, and
7. The risk-free rate, r, is constant and identical for all periods.

Calculating Black-Scholes Formula needs a comprehensive understanding of the volatility of an asset. The volatility, σ, is a measure of the uncertainty regarding an asset's returns. It described as the standard deviation of the return generated by the stock in 1 year when the return is stated using continuous compounding.

$$\frac{\delta S}{S} = \mu(\delta t) + \sigma\varepsilon\sqrt{\delta t} \qquad \text{Equation 3.1}$$

where:

$\frac{\delta S}{S}$ = Percent change of variable

$\mu(\delta t)$ = deterministic part of the change

$\sigma\varepsilon\sqrt{\delta t}$ = stochastic part of the movement

μ = drift term or growth factor

σ = volatility parameter

δt = time-steps

ε = simulated parameter

Black-Scholes formula combined underlying asset with normal distribution to calculate the option value (Equation 3.2).

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \qquad \text{Equation 3.2}$$

Where:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad \text{Equation 3.3}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \qquad \text{Equation 3.4}$$

c : call option value

S_0 : underlying asset

K : strike price

σ : asset volatility

r : continuously compounded risk free rate

T : time to maturity of the option

The normal distribution N(x) is a likelihood of a call option being executed in a risk-free environment. It is the cumulative probability distribution function for a variable or the probability of a variable will be less than a constant (x). In a risk-neutral universe, where stock prices less than the strike price are treated as zero. Furthermore, the expression $S_0N(d_1)e^{rt}$ is the predicted stock price at time T. The strike price is only paid if the stock price is greater than K, which has a chance of N(d2).

In the case study the option is defined as waiting to operate option. The company has right to operate, delay and abandon the project when the project is unprofitable. Respecting the multi uncertainty of cash flow, Black Scholes formula could not explain it. Under no investment assumption (K=0), Equation 3.2 will be error. However, the uncertainty is existing over project period. Therefore, the important of simulation method is to identify and assess the project value and the uncertainty when empirical approach cannot resolve it.

We employed BS model to the project with adaptation on the assumption. We add assumption of investment and focus only on the price uncertainty. The practice is done to be a benchmark to our proposed model later. We assumed an expenditure 1 Million USD to be strike price. Other assumption parameters are underlying asset of 7.514 MUSD, risk free rate of 0.25% and maturity time of 60 months.

Then, we model the asset uncertainty by measuring the volatility of the asset. We calculated the asset volatility as indicated by its standard deviation (Equation 3.5).

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \qquad \text{Equation 3.5}$$

Where:

σ : standard deviation

u_i : present value growth $= \ln\left(\frac{P_i}{P_{i-1}}\right)$

$\bar{u}$: mean of u_i

n+1 : number of observation

The calculation conducted further by applying price into cash flow model periodically (Figure 3.2). In Table 3.3, the calculated volatility of the project relating to price uncertainty was 29% in a monthly time frame.

$$\sigma_{project} = \sqrt{\frac{1}{60-1}(5.11)} = 0.29$$

Table 3.3 Project Volatility Estimation Based on Historical Tin Price

Period	**Tin, LME US$/t ($P_i$)**	**Project NPV (MUS$) ($C_i$)**	**Growth rate** (u_i)	$(u_i - \bar{u})^2$	$\sum(u_i - \bar{u})^2$
1/1/2016	14,906	$743.15	u_i= Ln(Pi/Pi-1)		5.11
2/1/2016	15,976	$1,395.45	0.63	0.35	
3/1/2016	16,729	$1,854.50	0.28	0.06	
4/1/2016	17,260	$2,178.21	0.16	0.02	
5/1/2016	16,324	$1,607.60	(0.30)	0.12	
6/1/2016	17,075	$2,065.43	0.25	0.05	
7/1/2016	17,840	$2,531.79	0.20	0.03	
8/1/2016	18,882	$3,167.03	0.22	0.04	
9/1/2016	20,164	$3,948.57	0.50	0.21	
10/1/2016	20,885	$4,388.11	0.11	0.00	
11/1/2016	21,320	$4,653.30	0.06	0.00	
12/1/2016	21,205	$4,583.19	(0.02)	0.00	
…	…	…	…	…	
…	…	…	…	…	
…	…	…	…	…	
1/1/2020	16,425	1,669.17	(0.24)	0.08	
2/1/2020	16,267	1,572.85	(0.06)	0.01	
3/1/2020	14,667	597.45	(0.97)	1.01	
4/1/2020	15,274	967.49	0.48	0.20	
5/1/2020	15,502	1,107.00	0.13	0.01	
6/1/2020	16,819	1,909.36	0.55	0.26	
7/1/2020	17,909	2,573.86	0.30	0.07	
8/1/2020	17,550	2,355.00	(0.09)	0.02	
9/1/2020	17,448	2,293.28	(0.03)	0.00	
10/1/2020	17,724	2,461.08	0.07	0.00	
11/1/2020	18,642	3,020.72	0.20	0.03	
12/1/2020	20,544	4,180.53	0.49	0.21	

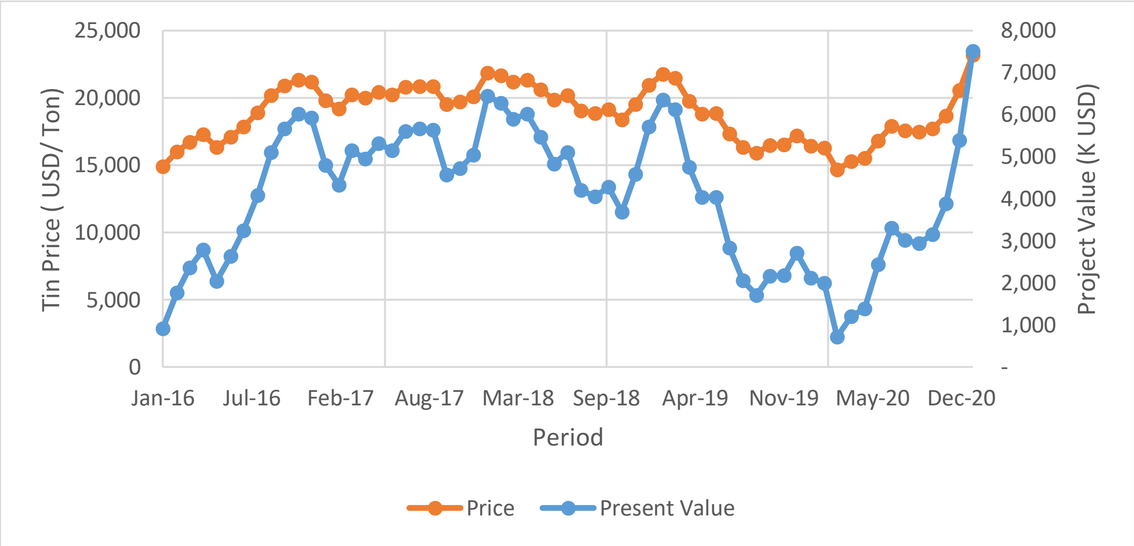

Figure 3.2 Price and Project Uncertainty

Thus, we input the parameter into formulation:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

$$c = 7.514 \cdot N(2.022) - 1\, e^{-0.25\% \cdot 60} N(-0.224)$$

c = 7.514 · 0.98 - 1 $e^{-0.25\% \cdot 60}$ 0.411

c = 6.997 MUSD

Where:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{7.514}{1}\right) + \left(0.25\% + \frac{0.29^2}{2}\right) \cdot 60}{0.29\sqrt{60}}$$

= 2.022

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= d_1 - \sigma\sqrt{T}$$

$$= 2.022 - 0.29\sqrt{60}$$

= -0.224

3.3. Binomial Lattice (BL) Valuation

Lattice valuation was founded by Cox et al., (1979) which simplified Black Scholes methodology. It started from the assumption that an asset follows a random walk. In each step, the asset has a certain probability of increasing and decreasing by a certain percentage amount. The approach followed a very important principle, namely risk-neutral valuation. When the time of lattice simulation is limited to zero, the value is equal to the result of the Black Scholes method. Recently, the lattice method has been popular in the academic research of real options analysis (Crundwell, 2008). Johnathan Mun described that the initial lattice approach of real option is the Brownian motion (Mun, 2006). The uncertainty in the future is explained by simulation. A likelihood distribution may be built at an increasing period based on all the virtual paths. The simulation pathways were developed using a Geometric Brownian Motion with set volatility defined at Equation 3.1.

In an assumption of no-arbitrage, an asset with S_0 price has an option with f value. During the T date, the stock can either move up to S_{0u}, where u >1, or down to S_{0d}, where d <1. In Figure 3.3, the option value will follow stock movement with f_u and f_d, respectively (Hull, 2015).

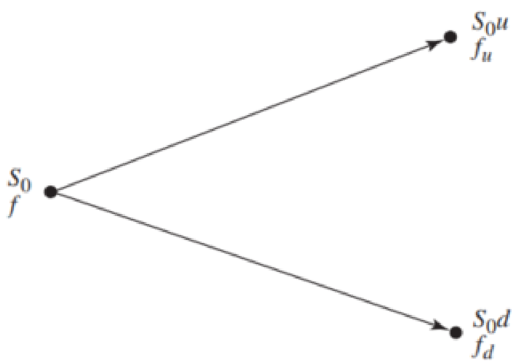

Figure 3.3 Stock and Option Value in One Period

The approach begins with measuring the growth of the project value using natural logarithm (ln) to address asset's volatility. The Brownian motion theory (Equation 3.1) can be derived to the exponential continue form (Equation 3.6) to account for the continuity of the option.

$$\frac{\delta S}{S} = e^{\mu(\delta t)+\sigma\varepsilon\sqrt{\delta t}} \qquad \text{Equation 3.6}$$

The upside factor (u) is when the simulated parameter generates a positive value or $e^{\sigma\sqrt{\delta t}}$. At the same time, down factor (d) is obtained when a negative value is produced or

$e^{-\sigma\sqrt{\delta t}}$. A manager should decide which maximizes the project value whether execution, delay or abandonment, to get the option value. Executing the project resulted company should pay the investing expenditure. In contrast, delaying and abandoning projects resulted the company expense or producing nothing. Furthermore, the probability of each way is calculated using risk-neutral probability. For instance, if the probability of upside is denoted as P, the expected value of a project is expressed at Equation 3.7. However, the project is conducted over time. Thus, the discount factor will be added in the exponential formula and generate Equation 3.8.

$$f_0 = P(f_{up}) + (1 - P)(f_{down}) \qquad \text{Equation 3.7}$$

$$f_0 = [P(f_{up}) + (1 - P)(f_{down})]e^{-r(t)} \qquad \text{Equation 3.8}$$

When assuming that the project is accepted when the initial value of asset (S_0) is minimal one. The probability can be defined as Equation 3.10.

$$P = \frac{e^{r\,(t)} - d}{u - d} \qquad \text{Equation 3.9}$$

Finally, after generating Probability (P), up factor (u), and down factor (d), the option value at present (f_0) can be obtained with Equation 3.8 (Crundwell, 2008).

The Lattice method is a simplification of Black Scholes formulation. Similar with Black Scholes formulation, the three uncertainty drivers cannot be accounted for by lattice valuation methodology. They adapted from financial option which written on an asset with certain exercise price with single uncertainty driver. The underlying asset for real option is the present value of cash flow while strike price is the initial investment. However, when the methodology applied to ongoing project which has no investment needed, they cannot calculate the uncertainty although uncertainty is always existing.

We added the assumption as we did at Black Scholes approach to compare Binomial Lattice (BL) method with other approach later. Then we applied the approach started calculated the up and down factor of 1.33 and 0.74. Following stage is calculating the up (P) and down (1-P) probability of 0.43 and 0.57. Finally, we made the tree model of the project value at Table 3.4.

Table 3.4 Project Value

Period (month)	0	1	2	3	...	60
project value (KUSD)	7,514	10,042	13,420	17,935	...	270,762,143,780
		5,622	7,514	10,042	...	151,599,282,030
			4,207	5,622	...	84,880,190,381
				3,148	...	47,524,279,948

Then, we subtract with the initial investment of 1 Million USD and we obtain Table 3.5.

Table 3.5 Project Value Minus Initial Investment

Period (month)	0	1	2	3	...	60
NPV (KUSD)	6,514	9,042	12,420	16,935	...	270,762,142,780
		4,622	6,514	9,042	...	151,599,281,030
			3,207	4,622	...	84,880,189,381
				2,148	...	47,524,278,948

Last step is backward calculation by applying Equation 3.8 and we got Table 3.6.

Table 3.6 Backward Calculation

Period (month)	0	1	2	3	...	60
Option Value (KUSD)	6,961	9,479	12,811	17,282	...	270,762,142,780
		5,074	6,984	9,464	...	151,599,281,030
			3,641	5,127	...	84,880,189,381
				2,526	...	47,524,278,948

3.4. Simulation Method

The simulation or numerical method is used when the analytic approach does not exist. It follows a stochastic process to predict the future. A stochastic process is defined as a variable whose value fluctuates unpredictably over time. Theoretically, stochastic modeling should satisfy Markov properties, Wiener, and Ito process. Markov properties is a form of a stochastic process explain that the present value is all that matters when forecasting the future. Since they are uncertain, future predictions must be described in terms of probability distributions. The Markov property states that the probability distribution of the price at any given future time is independent of the past price's path. At the same time, Wiener Process is a particular type of Markov stochastic process with a mean change of zero and a constant variance rate. The last property, the Ito process, is a generalized Wiener process. The parameters a and b are functions of the value of the underlying variable x and time t (Equation 3.10). Incorporating the three processes result in Equation 3.11.

$$dx = a(x,t)dt + b(x,t)dz \qquad \text{Equation 3.10}$$

$$\Delta x = a(x,t)\Delta t + b(x,t)\varepsilon\sqrt{\Delta t} \qquad \text{Equation 3.11}$$

This equation applies a small approximation. It presumes that the drift and variance rate of x remain consistent, equal to the values at time t, during the time interval between t and t+Δt. Note that the process in Equation 3.10 is Markov because the change in x at time t depends only on the value of x at time t, not on its history.

One way of gaining an intuitive understanding of a stochastic process for a variable is to simulate the behavior of the variable. The simulation involves dividing a time interval into many small time steps and randomly sampling possible variables' possible paths. The future probability distribution for the variable can then be calculated. A technique for sampling random outcomes for a stochastic process is named Monte Carlo simulation. Monte Carlo simulation has the benefit of being able to be employed when the payment is conditional on the path taken by the underlying variable S and when it is just dependent on the final value of S. However, Monte Carlo simulation has the disadvantage of being computationally intensive and unable to handle scenarios when early exercise opportunities exist (Hull, 2015).

Monte Carlo simulation employs the risk-neutral valuation outcome when valuing an option. The steps are:

1. Sample a random path for S in a risk-neutral simulation.
2. Calculate the payoff from the derivative.
3. Reproduce steps 1 and 2 to get multiple sample values of the payoff from the derivative in a risk-neutral simulation.
4. Compute the mean of the representative payoffs to estimate the expected payoff in a risk-neutral simulation.
5. Discount this expected payoff at the risk-free rate to get an estimate of the value of the derivative.

This thesis developed real option methodology particularly simulation method in mining industry. The mining industry has several uncertain variables during production. In this study, an evaluation method based on Monte Carlo Simulation was developed for economic assessment through real option (RO) analysis, namely Stratified State Aggregation (SSA). The methodology benefits the simplicity and flexibility to incorporate all uncertainties in one assessment. In the oil industry, SSA assessment based on simulation has been performed

(Adachi et al., 2008). However, none of them is applied in the mining project, which has different calculation methods on uncertainties calculation. The considerable risk and uncertainties in this study are economic, technical, and geological factors represented by price, cost, and reserve, respectively. However, modeling the uncertainty is unique, and we developed it through this research. Nevertheless, integration of those uncertainties will improve the applicability of RO in the mining industry. Detail work of simulation approach is explained at Chapter 4.

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Chapter 4 Multi-stage Stratified State Aggregation

4.1. Stratified State Aggregation (SSA)

Stratified State Aggregation (SSA) algorithm was a systematic numerical technique to value American options. It was able to compute the prices of complex American instruments in seconds on a workstation. The algorithm was aimed to resolve the problem of the classical method to overcome high dimensional valuation. The methodology applied Monte Carlo Simulation for modeling uncertainty and performed a backward calculation to get the value at present (Barraquand & Martineau, 1995).

Multi-stage SSA (MSSA) is an RO simulation method which run through Java programming. Developed MSSA algorithm was used for the oil industry with price uncertainty driver (Adachi et al., 2008). It was developed to analyze oil development projects, including early stages of exploration, development, and production. Every stage had its capital expenditure, and the success probability increased with the expenditure. They calculated the value of a crude oil development project using real options analysis in a multi-stage investment with geological and pricing uncertainties that decreased as the project progressed. The research approached investment initiatives as a multi-staged compound or rainbow option investment akin to a research and development project. Pricing uncertainty and diminishing geological uncertainty were accounted for as project values vary overtime to find the best moment to go from exploration to development and eventually extraction.

This research performed identical Java scripts in mining areas which has different properties with the oil industry, especially uncertainty modeling. We practiced on developed mining site, especially underwater tin mine planning. The company has the right to extract the mineral considering the uncertainty of selling price, operating cost, and the amount of mineral itself. Regarding the limited number of dredges as main equipment, every mining location is treated as stage extraction. Therefore, MSSA is an alternative method to compute the real value of multi project with multi uncertainties driver.

MSSA method started with building stratification map, which represents a partition or state-space path traveled by the asset's value. It was separated into thin layers along with the maps like the optimal stochastic control path shown in Figure 4.1. The controlling path was price, grade, and cost uncertainty.

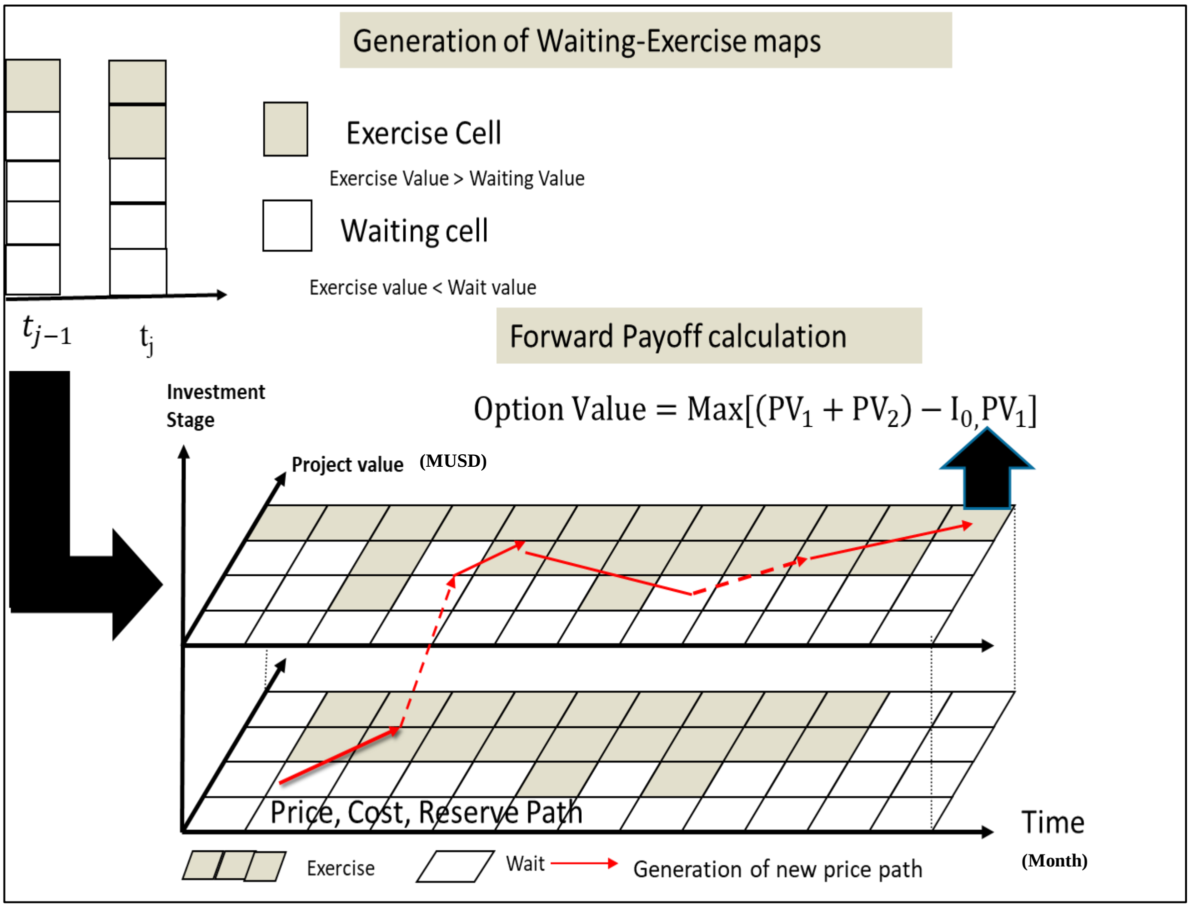

Figure 4.1 Multi-stage Stratified State Aggregation Methodology

The algorithm simulates project value 100,000 times considering three uncertainties. The method is an alternative to combine three uncertainties and more than one project. In the case study, the option type calculation is delay option of two mining project. The company has the right to mine, wait or abandon the project considering the three uncertainties.

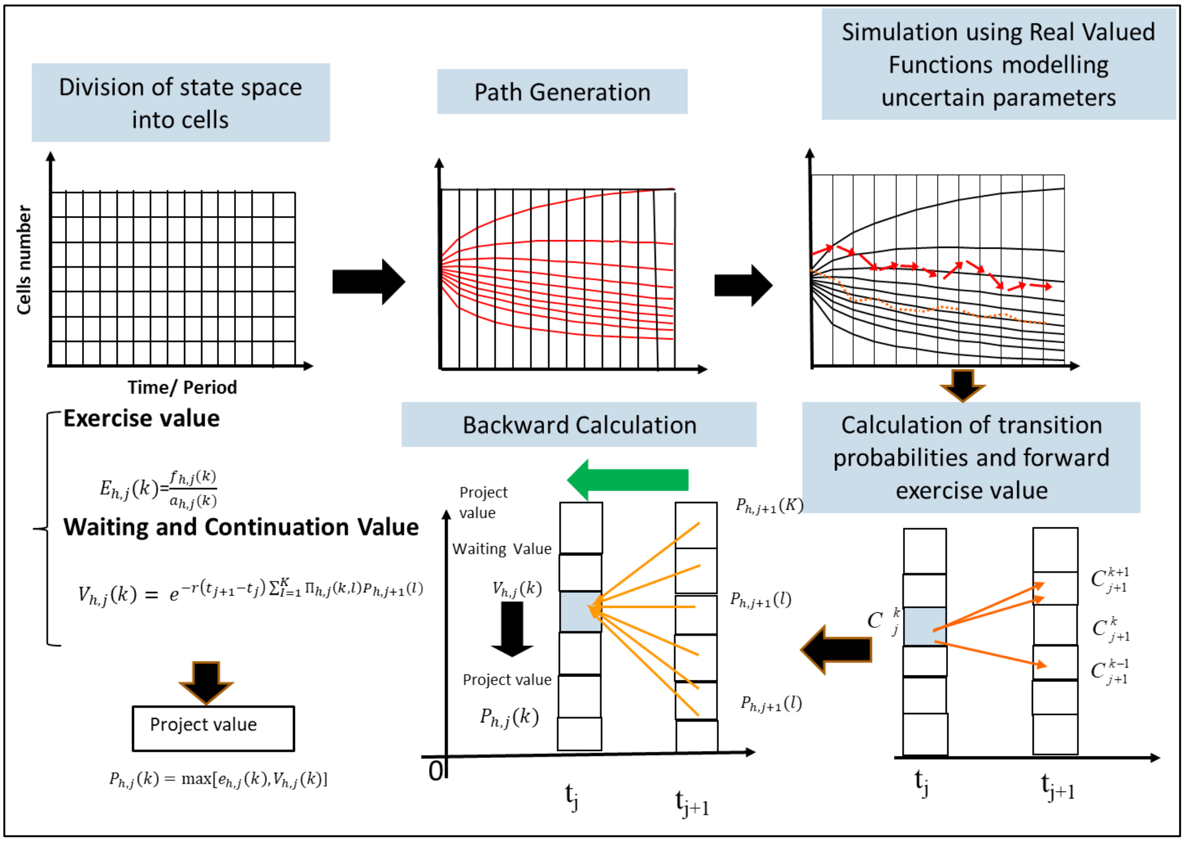

Figure 4.2 Stratified State Aggregation (SSA) Methodology

Figure 4.2. detailed the steps of the MSSA algorithm. The algorithm was started by building 400 cells as a state-space of the project value. The multi-stage development was defined according to the parameters S.

$S^{a}_{h,j}$ (i) : the value of the asset a (a=1,….A) at time t_j to maturity in the i^th^ path of stage h

$S_{h,j}$= ($s^{1}_{h,j}$ (i), $S^{2}_{h,j}$ (i),….$S^{A}_{h,j}$(i)): the i^th^ mapped path of multiple assets in stage h

The state-space path was divided to run through several cells (cell division stage) corresponding to the average expected payout of the total number of pathways.

Then, whole range of $S_{h,j}$ was divided into intervals $C^{k}_{h,j}$

$C^{k}_{h,j}$ = $[c^{k}_{h,j}, c^{k+1}_{h,j}]$ (k = 1,…K)

$S_{h,j}$ = in the state k when $S_j \in C^{k}_{h,j}$

$P_{h,j}$ (k) = the exercise value of the investment option at t_j in stage h

$V_{h,j}$(k) = the continuation value when the option is not exercised at t_j in stage h

Furthermore, the division path of project value was generated using Monte Carlo simulation to model uncertain parameters. Generated the mapped paths of the asset $S_{h,j}$ (i) and counted the number of paths $a_{h,j}$(k) such that $S_{h,j} \in C^k_{h,j}$ (Equation 4.1).

$$S_{h,j}\ (i) \in C^k_{h,j}\ \cap S_{h,j+1}$$

$$f_{h.j}\ (k) = \begin{cases} \sum_{S_{h.j} \in C^k_{h,j}} (e^{-rU_h}\ P_{h+1,j+U_h}(m) - I_o) \\ \sum_{S_{h,j} \in C^k_{h,j}} (e^{-rU_h}\ S_{h,j+U_H} - I_o) \end{cases} \qquad \text{Equation 4.1}$$

Where

r : Risk free interest rate

I_0 : Exercise price (investment) in stage h.

$S_{h,j+U_h} = S_{h+1,j+U_h} \in C^m_{h+1,j+U_h}$ when h = 1,……,H-1

Next step was calculating transition probabilities from $C^k_{h,j}$ to $C^m_{h,j+1}$ and forward exercise value $e_{h,j}(k) = \frac{f_{h,j}(k)}{a_{h,j}(k)}$ (Equation 4.2). Finally, we maximized exercise value and waiting value (Equation 4.3).

$$\prod\nolimits_{h,j}(k.k+1) = \frac{\text{Number of paths from } C^k_{h,j} \text{ to } C^{k+1}_{h,j+1}}{\text{Number of paths through } C^k_{h,j}} \qquad \text{Equation 4.2}$$

$$P_{h,j}(k) = \max[E_{h,j}(k), V_{h,j}(k)]$$

Where: $V_{h,j}(k) = e^{-r(t_{j+1}-t_j)} \sum_{I=1}^{K} \prod_{h,j}(k,l) P_{h,j+1}(l)$ Equation 4.3

$$E_{h,j}(k) = \frac{f_{h,j}(k)}{a_{h,j}(k)}$$

As long as the value of waiting to invest is higher than the value of investing immediately, the company should defer the investment decision (Mun J., 2006). Otherwise, they do not need to wait to decide to invest or abandon the project.

4.2. Economic Uncertainty Model

Modeling uncertain parameters must follow several principles of stochastic calculus. A stochastic process is a variable that changes over time in a way that is at least in part random. Formally, it is defined by probability law for evolution variable (x) through time (t). The model must satisfy Markov processes which are described as later variable estimation (x_{t+1}) depending only on previous data (x_t) and not additionally on condition before time t (Dixit & Pindyck, 1993).

Price is a time series variable in the cash flow parameter. Therefore, forecasting price and modelling the uncertainty should be started by statistical analysis of historical price. One of the analyses is unit root test to check the stationary data. The most popular one, and the one that we used, is the Augmented Dickey–Fuller (ADF) test. This test is based on the process $y_t = \rho y_{t-1+} v_t$ is stationary when $\rho < 1$, but when $\rho=1$, it becomes the nonstationary random walk process $y_t = \rho y_{t-1+} v_t$. Hence, one way to test for stationarity is to examine the value of ρ. In other words, we test whether ρ is equal to one or significantly less than one. where the v_t are independent random errors with zero mean and constant variance δ^2. Non stationary data can be modelled using random walk model or Geometric Brownian Motion (GBM) model (Hill et al., 2011).

The tin price is collected from January 2016 to December 2020 to model the price in 5 years' production. The ADF test resulted the data was not stationary (Table 4.1). The null hypothesis that tin price has a unit root can be rejected because the error probability was 27% or more than 5% threshold. Therefore, we used Geometric Brownian Motion (GBM) model to simulate SSA. Brownian motion model is a continuous-time stochastic process with three important properties: Markov process, Wiener process (probability distribution is independent of any other time interval), and normally distributed with increasing variance over time.

Table 4.1 ADF test on Tin Price data (2016-2020)

Null Hypothesis: TINPRICE has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.032564	0.2725
Test critical values:	1% level	-3.546099	
	5% level	-2.911730	
	10% level	-2.593551	

*MacKinnon (1996) one-sided p-values.

Further uncertainty modelling stage is measuring the volatility of the asset. Thus, we analyze the price volatility as indicated by its standard deviation (Equation 4.4). The price is frequently watched at defined periods to assess the volatility of a price experimentally.

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \qquad \text{Equation 4.4}$$

Where:

s : standard deviation

u_i : price growth $= \ln\left(\frac{P_i}{P_{i-1}}\right)$

$\bar{u}$: mean of u_i

n+1 : number of observation

The standard deviation of the u_i is $\sigma\sqrt{\tau}$, where τ is the length of time, thus, σ=s/√τ. It is not easy to pick a fair value for *n*. More data typically leads to greater accuracy, although accuracy varies with time, and too old data for predicting future volatility may be irrelevant. Alternatively, *n* can be specified as the number of days to which volatility will be applied as a rule of thumb (Hull, 2015).

Mining project revenue depends on the price of sold material. In the case study, historical price sourced from S&P data from January 2016 to December 2020. The unit root analysis through Augmented Dickey Fuller test resulted a nonstationary data with probability 27%. Therefore, we used GBM assumption to model the price in the project life. Application of Equation 4.4 to tin price (Table 4.2) resulted standard deviation of 4.7%. The fluctuation was based on 60 monthly data. The volatility will be used to predict 5 years later.

$$s_{price} = \sqrt{\frac{1}{60-1}(0.1373)}$$

$$s_{price} = 0.047$$

Table 4.2 Tin Price Volatility

Period	**Tin, LME US$/t ($P_i$)**	**Growth rate** (u_i)	$(u_i - \bar{u})^2$	$\sum(u_i - \bar{u})^2$
1/1/2016	14,906	u_i= Ln(Pi/Pi-1)		0.1373
2/1/2016	15,976	0.07	0.0043	
3/1/2016	16,729	0.05	0.0018	
4/1/2016	17,260	0.03	0.0007	
5/1/2016	16,324	(0.06)	0.0036	
6/1/2016	17,075	0.04	0.0017	
7/1/2016	17,840	0.04	0.0016	
8/1/2016	18,882	0.06	0.0028	
9/1/2016	20,164	0.07	0.0038	
10/1/2016	20,885	0.04	0.0010	
11/1/2016	21,320	0.02	0.0003	
12/1/2016	21,205	(0.01)	0.0001	
...	...	...	...	
...	...	...	...	
...	...	...	...	
1/1/2020	16,425	(0.04)	0.0027	
2/1/2020	16,267	(0.01)	0.0003	
3/1/2020	14,667	(0.10)	0.0123	
4/1/2020	15,274	0.04	0.0011	
5/1/2020	15,502	0.01	0.0001	
6/1/2020	16,819	0.08	0.0055	
7/1/2020	17,909	0.06	0.0031	
8/1/2020	17,550	(0.02)	0.0008	
9/1/2020	17,448	(0.01)	0.0002	
10/1/2020	17,724	0.02	0.0001	
11/1/2020	18,642	0.05	0.0019	
12/1/2020	20,544	0.10	0.0081	

Logarithmic natural growth analysis based on historical data resulted in the price has an average monthly growth rate (α) of 0.74% at Table 4.2 and detailed ata Appendix A. However, to be applied at our evaluation method, we assumed the price growth would follow the stochastic differential equation model, particularly the one-factor constant convenience yield model for risk-neutral prices stated in Equation 4.5 (Cortazar et al., 2001). Assumed parameters were risk-free rate (r) of 0.25%, convenience yield (δ) 0.17% and calculated price volatility (σ_P) of 4.7%. We simulated the GBM price forecasting with the assumed parameter at Figure 4.3.

$$\frac{dP}{P} = (r\text{-}\delta)dt + \sigma_P dz_P \qquad \text{Equation 4.5}$$

Where P = price,

r = risk free rate,

δ = convenience yield,

σ_P= price volatility,

dz_P = Wiener Process.

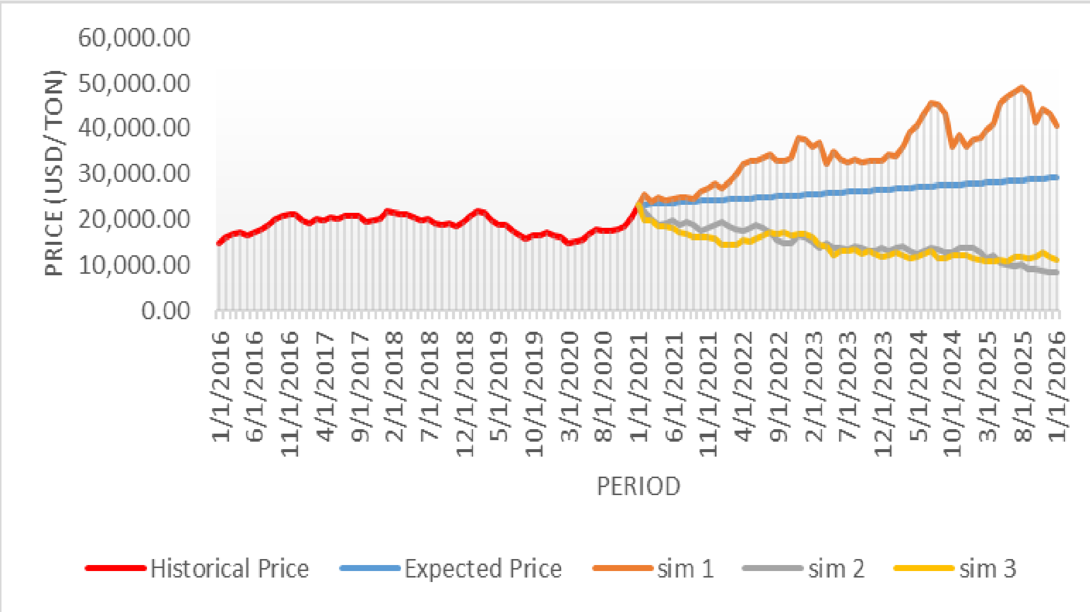

Figure 4.3 Price Simulation Model (Geometric Brownian Model)

Economic uncertainty is represented by price fluctuation. MSSA approach is an alternative to evaluate that risk. With the assumption of no added investment and price uncertainty, the project should continue to operate in January 2021. The price of tin per ton was 23,199 USD in January 2021. It had a net present value (NPV) of 7.5 MUSD, real option value (ROV) of 7.68 MUSD, and a 1.99% option premium below our 5% threshold. The critical present value of 3 MUSD was reached when the tin price dropped 25% to 17,399 USD/ton (Figure 4.4). The condition altered the option premium to 8%, and the company should delay the project. Moreover, the decreasing price of 45% into 12,759 USD/ton made option value reach zero, and consequently, the project is completely unprofitable.

The variability of real option value can only be captured when we simulated varies of the initial investment in either Black Scholes or lattice valuation. An ongoing project can only be evaluated through the simulation method. The underlying asset represented by present value could not capture permutation between cash flow variable which MSSA approach could do.

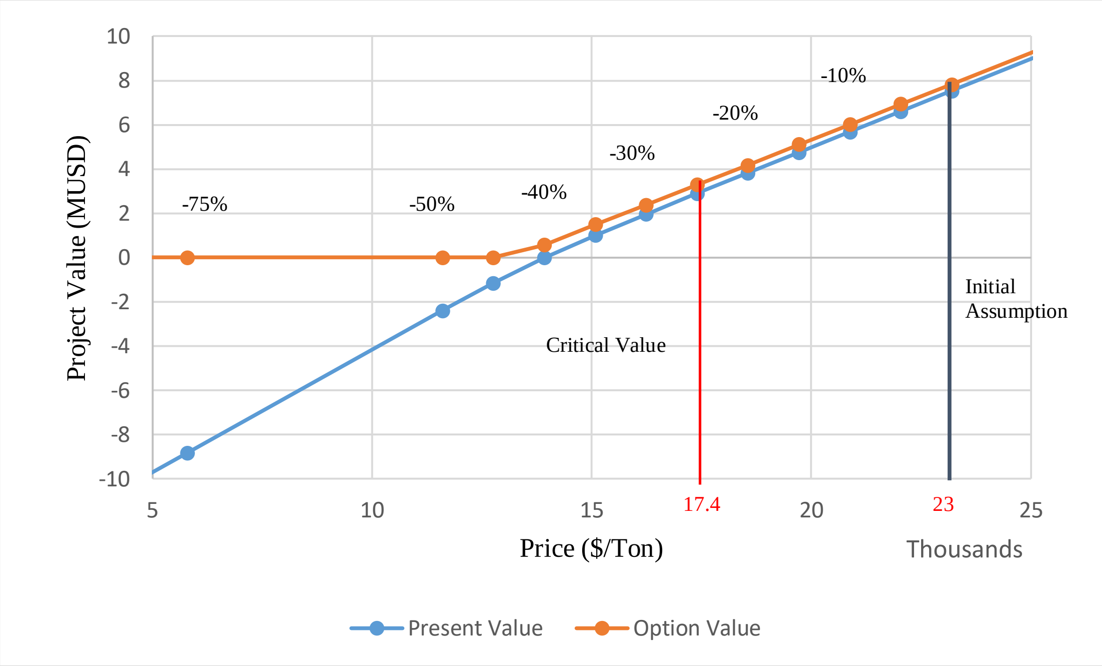

Figure 4.4 Project Value Considering Price Changing only

Conducting RO analysis through common approach is needed to benchmark proposed method. As a result, we assumed a capital cost of 1 MUSD and considering price uncertainty only. We conducted on the 3 approaches of Black Scholes (BS) Formulation, Binomial Lattice (BL) valuation, and Stratified State Aggregation (SSA) methodology. Asset or project uncertainty is used to calculate real option value through Black-Scholes and Lattice method.

The comparison result is resumed at Table 4.3. It concluded that the differences between approaches are not significant. However, the SSA has advantages on the calculation on multi uncertainty and multi stage of project. The only disadvantage of SSA method is calculation time of the number simulation. In the case study, we used 100,000 simulations to get optimum number of simulation.

Table 4.3 Comparison of Real Option Approach

Parameters	SSA Value	BL Value	BS Value	Unit
Initial Project Value	7.54	7.54	7.54	MUSD
Exercise Price	1.0	1.0	1.0	MUSD
Price uncertainty value	4.7%/month	4.7%/month	4.7%/month	
Option Value	6.921	6.960	6.997	MUSD
Option premium	6.8%	7.6%	8.2%	
Decision	Delay	Delay	Delay	

4.3. Geological Uncertainty Model

Reserves have varying levels of geoscientific confidence and uncertainty. A mining company must spend much capital to get confidence in geological uncertainty. Geological uncertainty is a spatial uncertainty that revealed after the reserve is extracted. Typically, kriging methodology is used to estimate the total amount of reserves. However, it could not be enough to capture the uncertainty. Therefore, we used Conditional Simulation (CS) method to measure the geological uncertainty. It is a developed methodology based on the kriging method (Dowd, 1994).

The valuation of mining projects is based on conditional simulation (CS) of ore body parameters. CS is a geostatistical tool that can generate punctually or block 'realizations' of mineral grades. Each realization is intended to honor the histogram and semivariogram of the true grade distribution and honor known data points (Nicholas, 2014). It is a technique to measure spatial uncertainty based on spatial Monte Carlo Analysis (MCS). CS reproduces the properties of an ore body and quantifies the variability, which we call "spatial uncertainty." The realization of CS methodology on our case study is figured at Figure 4.5. The utilization of MCS is for the NPV probability distribution function and the variability of the projected cash flow (Jurdziak L and Wiktorowicz, 2008).

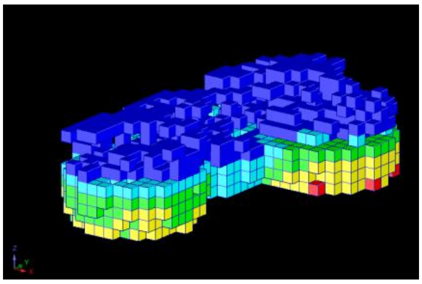

Figure a Conditional Simulation 1

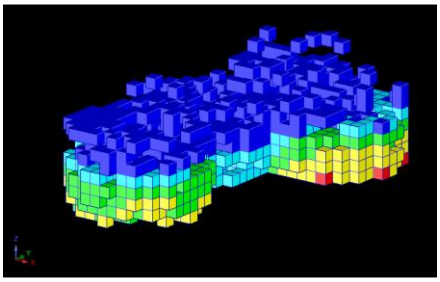

Figure b Conditional Simulation 1000

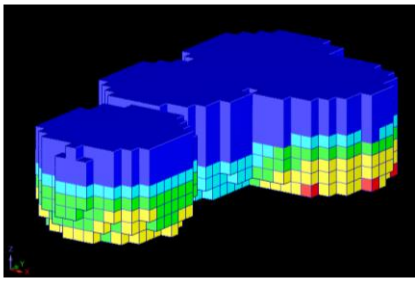

Figure c Expected Conditional Simulation

r=0.00 g=0.00 b=1.00	0.00 -> 0.50
r=0.00 g=1.00 b=1.00	0.50 -> 1.00
r=0.00 g=1.00 b=0.00	1.00 -> 1.50
r=1.00 g=1.00 b=0.00	1.50 -> 2.00
r=1.00 g=0.00 b=0.00	2.00 -> 2.50

Figure 4.5 Conditional Simulation for Modelling Reserve

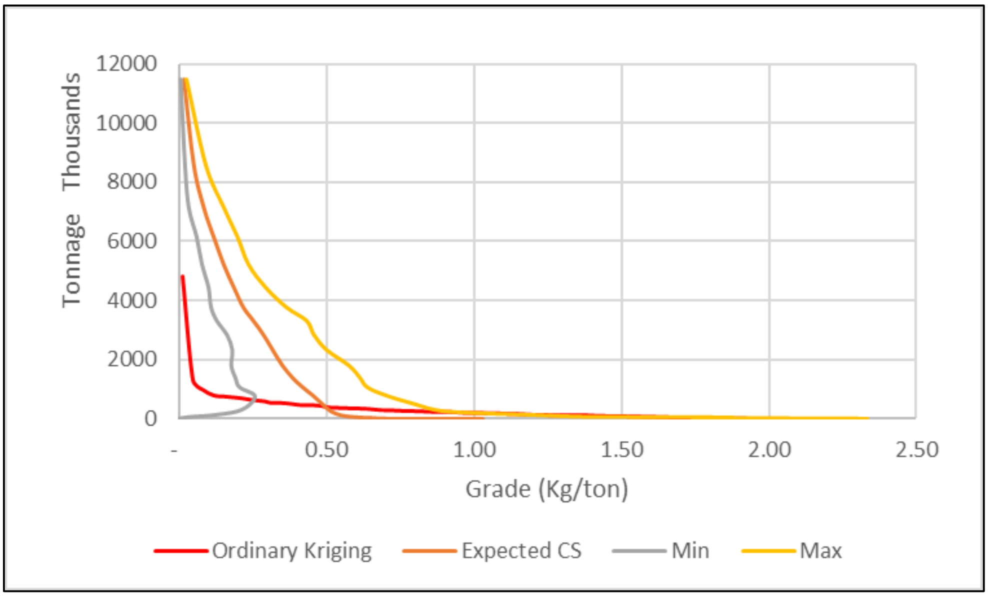

Figure 4.6 Mid Reserve Grade-Tonnage

We found that each location of resource has its expected grade and tonnage as well as volatility (Figure 4.6 and Figure 4.9). In our 1000 simulation, we get reserves amount 7.2 Mton, grade 0.594 volatility 9% (Figure 4.6, and Figure 4.7) and grade amount 1.1 Mton, grade 0.558, volatility 15.8% (Figure 4.9, and Figure 4.10) for mid and south reserve, respectively. The stability of the result is reached after more than 400 simulations. The probability distribution of Conditional Simulation result followed normal distribution (Figure 4.8 and Figure 4.11). Thus, we concluded the result is reliable to be the base of economic evaluation using discounted cash flow analysis and real option valuation methodology.

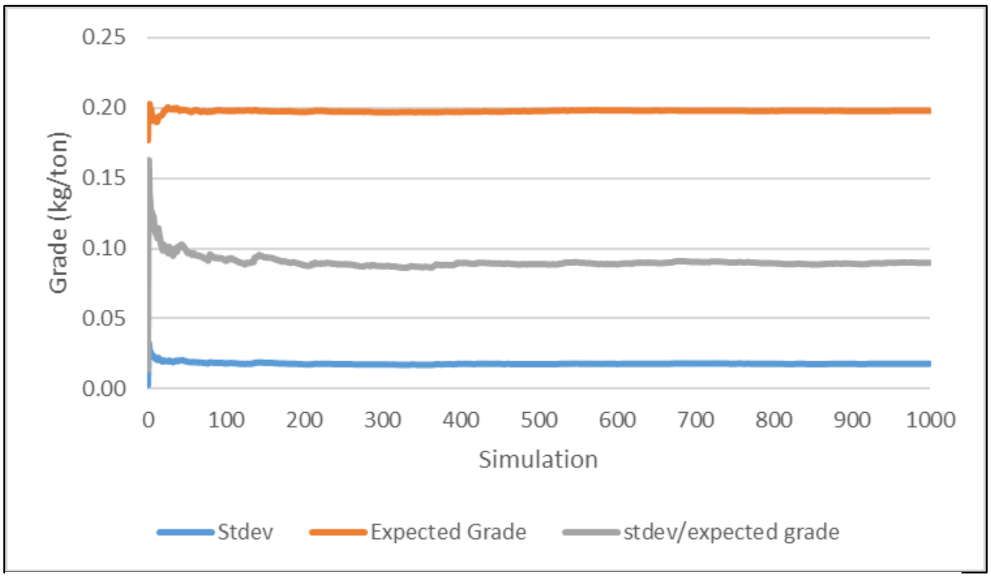

Figure 4.7 Mid Grade Uncertainty

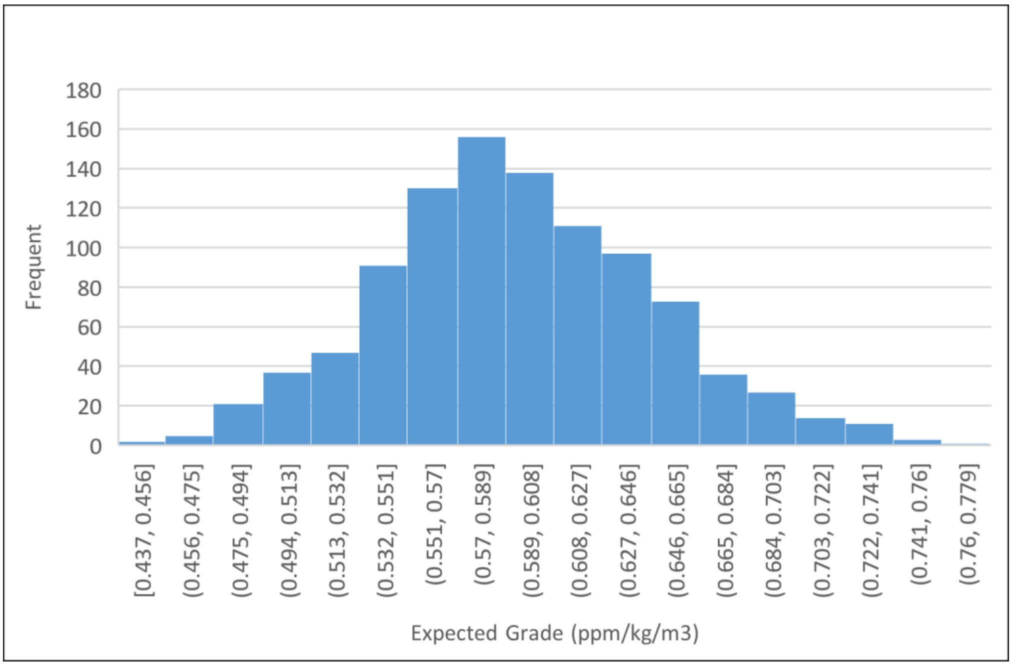

Figure 4.8 Probability Distribution (Mid Reserve)

In the research, one originality is the utilization of conditional simulation results, which are typically only for estimating reserve. While, herein, we used to calculate the grade uncertainty and calculate further the project value through real options analysis. The common Kriging method could not provide variability of the reserve. Therefore, conditional simulation was alternative method to calculate grade uncertainty and anticipate it in a project lifetime.

Geological uncertainty was represented by the grade uncertainty, explained in the previous chapter. The originality of this research was the utilization of conditional simulation to be combined with real options methodology. It can only be measured using MSSA methodology and resulted in Figure 4.10. The result of geological uncertainty measurement showed a similar outcome with economic risk. The reason was with a similar modeling method, which belongs to the Geometric Brownian Model and the modeling at income sector.

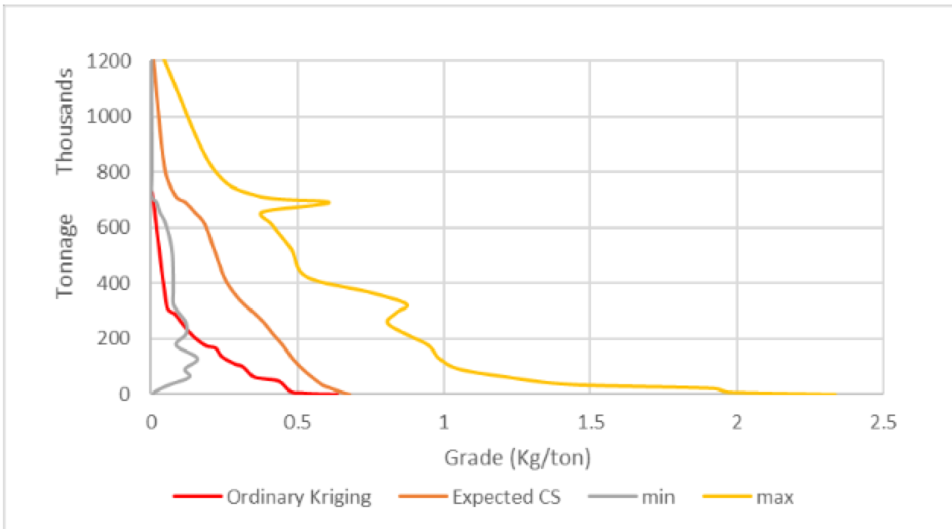

Figure 4.9 South Reserve Grade-Tonnage

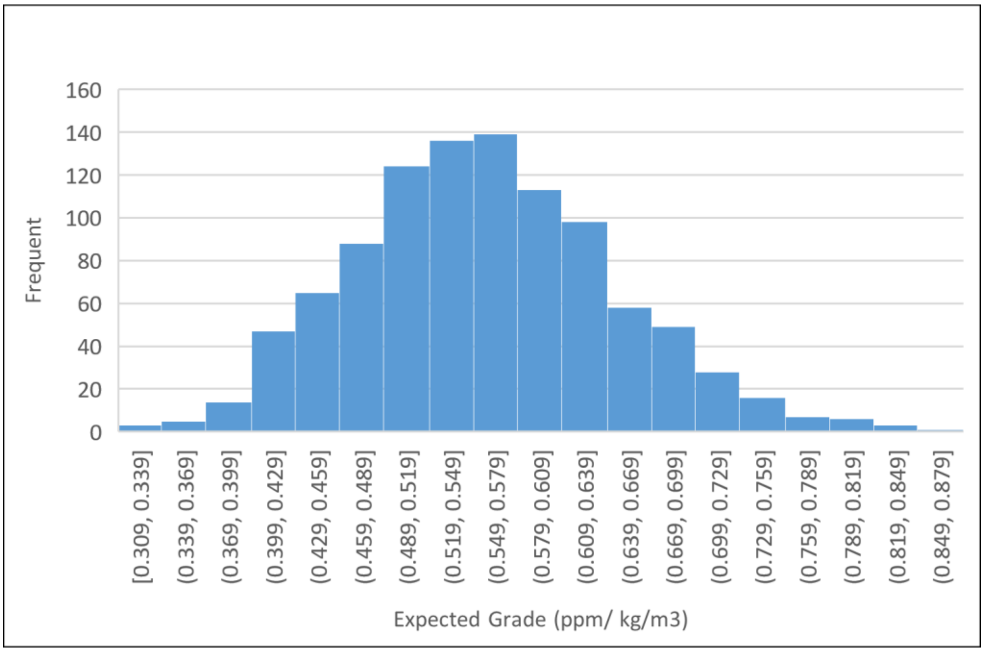

Figure 4.10 Probability Distribution (South Reserve)

Economic (price) uncertainty had positive growth assumptions while geological risk modeling assumpted only followed normal distribution assumption. The calculation through

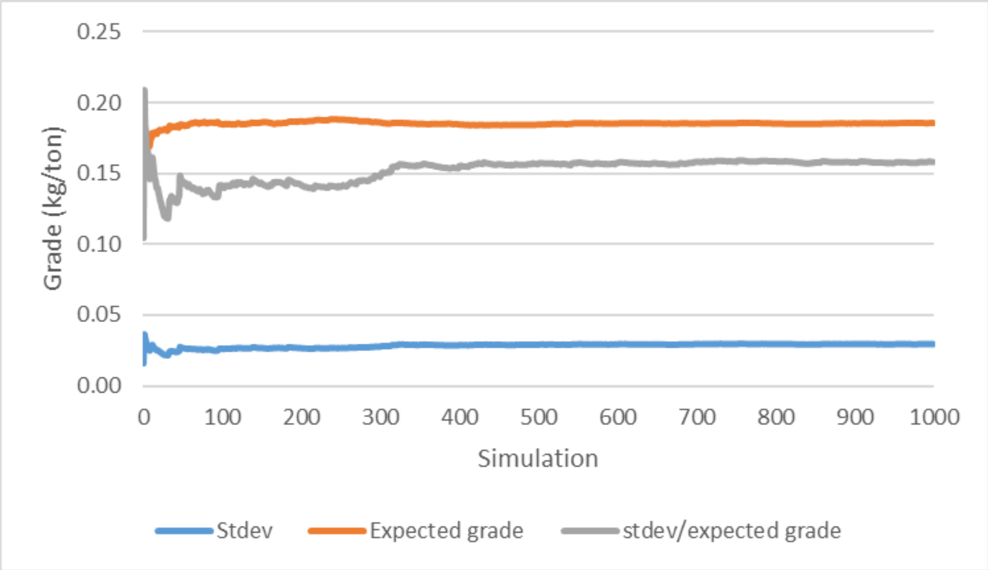

Figure 4.11 South Grade Uncertainty

SSA approach had a net present value (NPV) of 7.5 MUSD and real option value (ROV) of 7.63 MUSD (Figure 4.12). SSA method replicated the distribution of conditional simulation and multiply the probability and the value of each simulation. It indicated an option premium of 1.29% and recommended continuing the project. When the reserve is dropped 25%, the option premium reached 9%. It surpassed our 5% threshold and recommended delaying the project. Furthermore, when the grade is dropping 40%, the decision recommendation shifted to postpone the project.

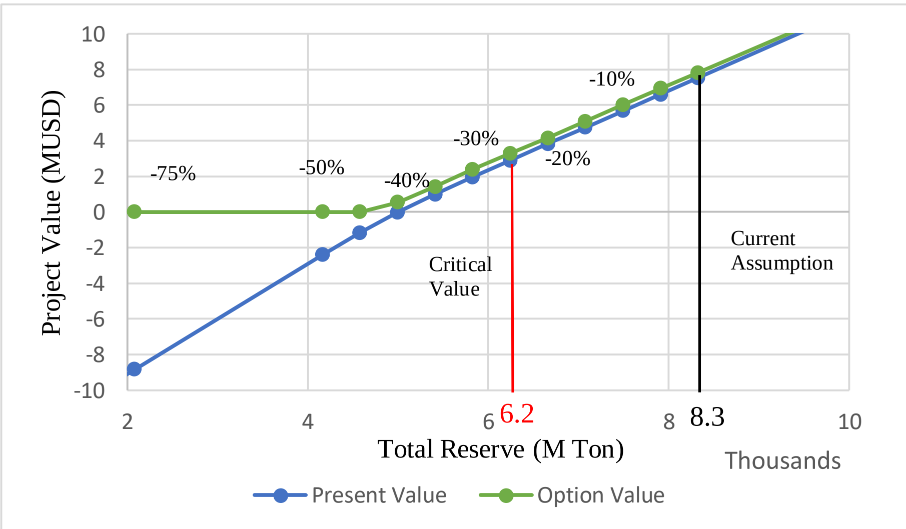

Figure 4.12 Project Value Considering Grade Changing only

4.4. Technical Uncertainty Model

We measure the technical uncertainty represented by cost volatility. The cost consisted of mining, processing and metallurgical cost. Unit cost of production model construction was based on historical cost given by the company with a similar location and production technique (Figure 4.13). Similar ADF methodology was applied to the summarized historical unit cost data (Figure 4.14). The unit root test concluded that the cost data is stationer (Table 4.4). Therefore, the construction of the technical uncertainty model was used the Mean Reverting (MR) model (Dixit A.K. and Pindyck R.S, 1993) instead of the GBM model. We used the simplest mean-reverting process-also known as the Ornstein-Uhlenbeck process (Equation 4.8). The process follows Markov property and does not have independent increments.

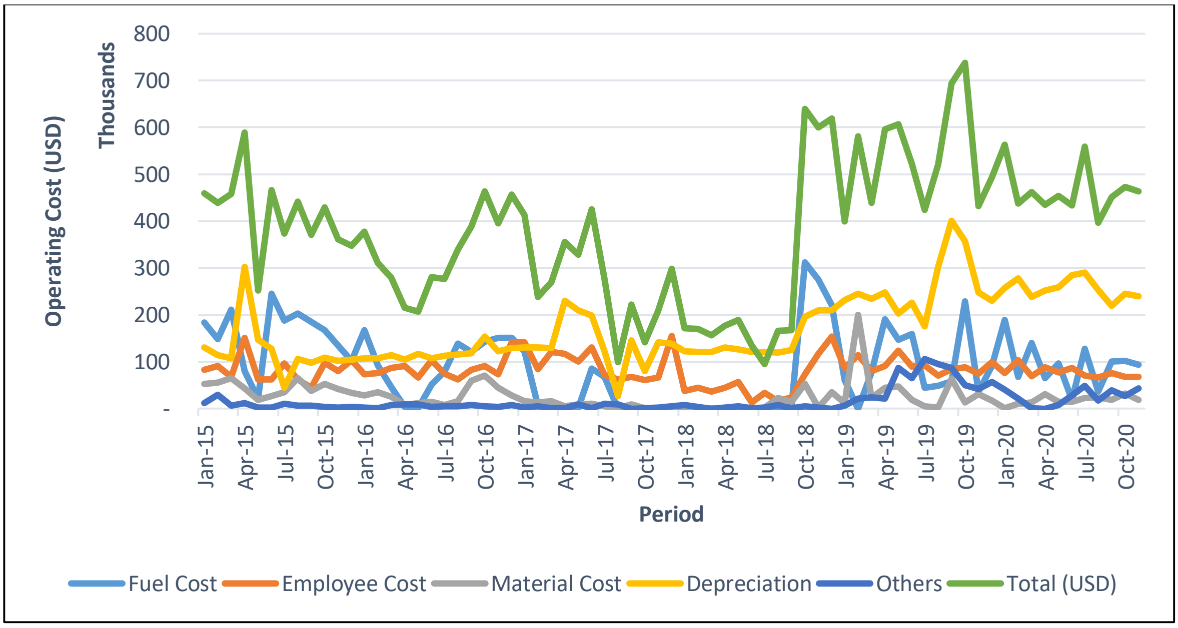

Figure 4.13 Historical Operating Cost Data

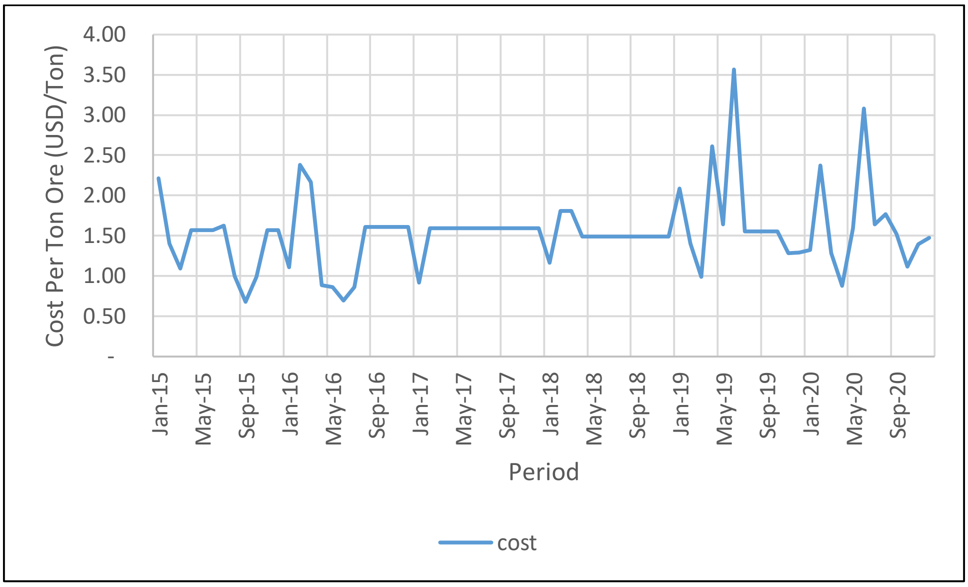

Figure 4.14 Modified Historical Unit Cost Data

Table 4.4 ADF Test of Cost data

Null Hypothesis: COST has a unit root			
Exogenous: Constant			
Lag Length: 0 (Automatic - based on SIC, maxlag=10)			
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.633826	0.0000
Test critical values:	1% level	-3.544063	
	5% level	-2.910860	
	10% level	-2.593090	
*MacKinnon (1996) one-sided p-values.			

Parameters calculation for the MR process started by running discrete-time data regression (Equation 4.6).

$$x_t - x_{t-1} = a + b\, x_{t-1} + \varepsilon_t \qquad \text{Equation 4.6}$$

Then, we calculated x = -a/b, and $\eta = -\log(1 + b)$, and finally, we calculated the standard error of the regression using equation 4.7.

$$\sigma = \sigma_\varepsilon \sqrt{\frac{\log(1+b)}{(1+b)^2 - 1}} \qquad \text{Equation 4.7}$$

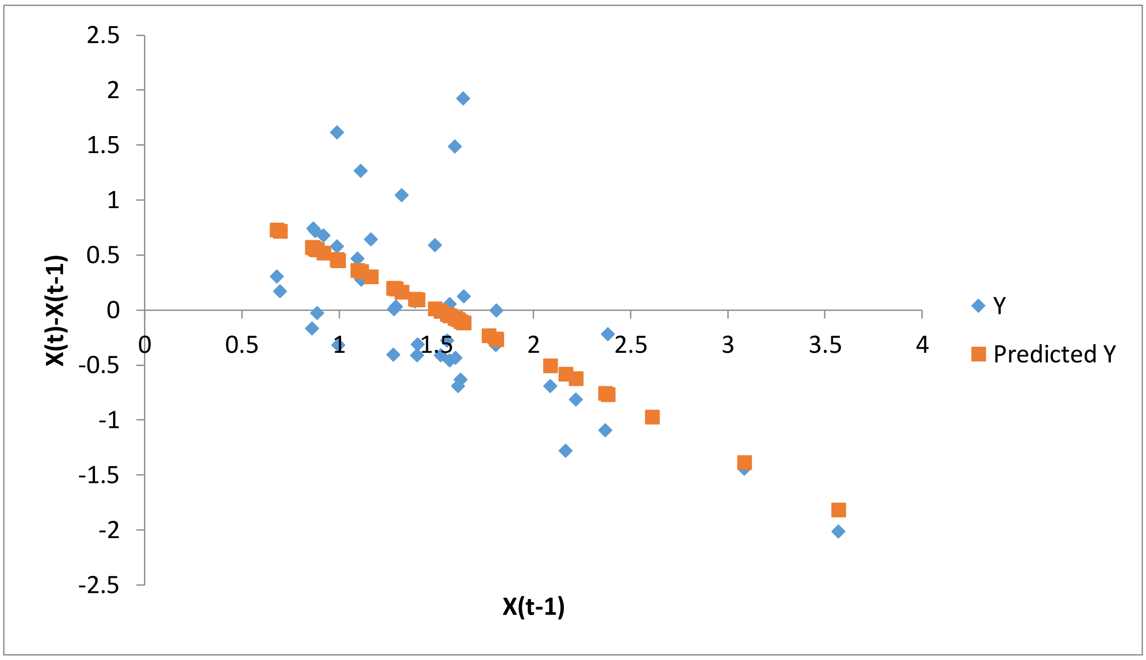

Figure 4.15 Regression Analysis of Historical Cost

The regression model in Figure 4.15 resulted in intercept (b) 1,33 and variable (a) (-0.88). In addition, it has an R squared error of 44%. Although the accuracy is quite low, the simulation shows similarity with the historical data. It has better results than the GBM model (Figure 4.16). In the Figure 4.16, the data was real historical value before January 2021and followed by the prediction. In summary, the linear regression cost model tends to be back to its mean in 92% rate (η). Nevertheless, the volatility σ_c rate is very high, reaching 39%.

$$\frac{dC}{C} = \eta(\bar{x} - x)dt + \sigma_c dz_c \qquad \text{Equation 4.8}$$

In our research, technical uncertainty is related to the cost of operation, which consists of mining cost, processing cost, and metallurgical cost. The purpose is to compare with other uncertainty and the strategy to hedge the uncertainty. Historical data analysis showed that the uncertainty is very high, reaching 39%. Additionally, the model had a different approach with other uncertainties, which used the Mean Reverting Process and affected the expense side of the project. Increasing cost decreased project value and vice versa. With the cost uncertainty being the only driver as assumptions, the project is recommended to be delayed with Net Present Value (NPV) 7.5 MUSD and Real Option Value (ROV) 15 MUSD (Figure 4.17). It had an option premium of 100% far beyond our assumed threshold. The option premium can be pressed below our threshold when either the cost dropped 75% or the volatility decreased to 5%.

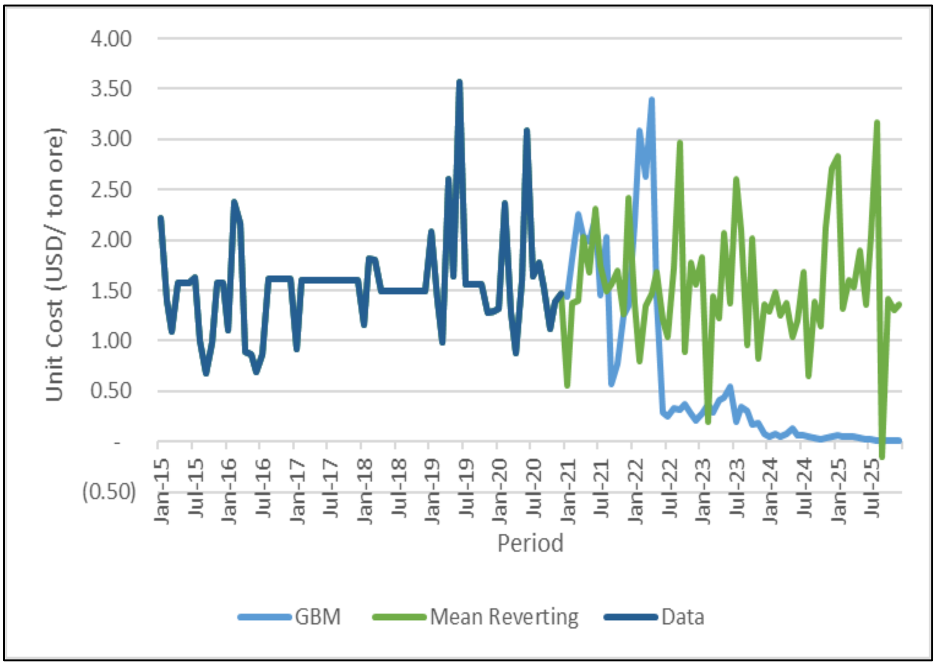

Figure 4.16 Cost Uncertainty Model (GBM vs MR)

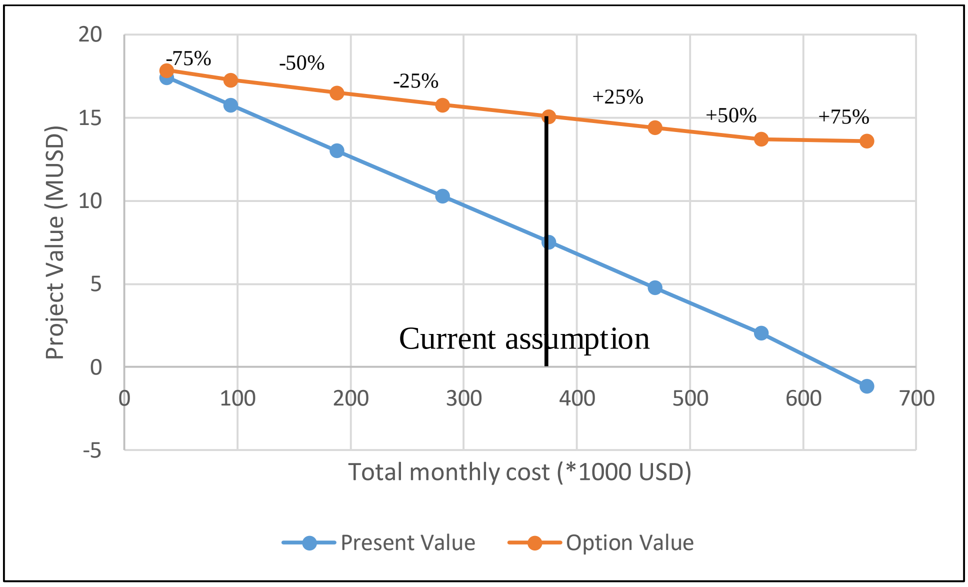

Figure 4.17 Project Value Considering Cost Changing only

4.5. Combined Uncertainty Assessment

Price, grade, and cost uncertainties were the driver of project risk assessment. The mining project resulted in net present value output with SSA method in Java was 7.5 MUSD and real option value 17.6 MUSD. Generated, exercise, and waiting cells values as illustrated in Figure 4.18, Figure 4.19, and Figure 4.20., respectively. They were randomly picked simulations from both locations of mining. The exercise values were the overall payoff if one decides to invest. The waiting values were the expected overall payoff if one decides not to

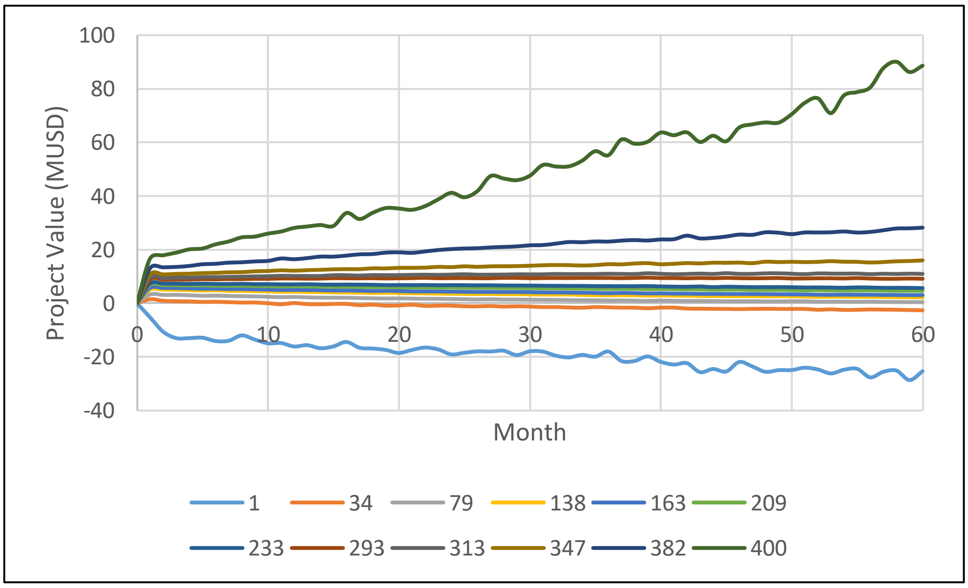

Figure 4.18 Generated Cells with Three Uncertainties Driver (Random Cells Picked)

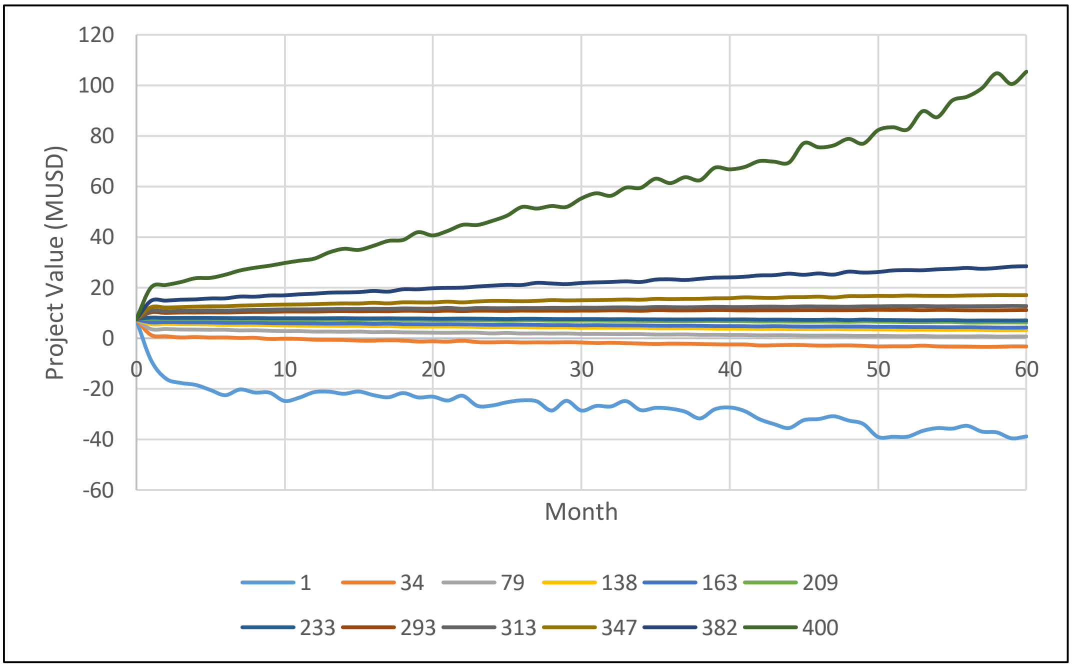

Figure 4.19 Exercise Value with Three Uncertainties Driver (Random Cells Picked)

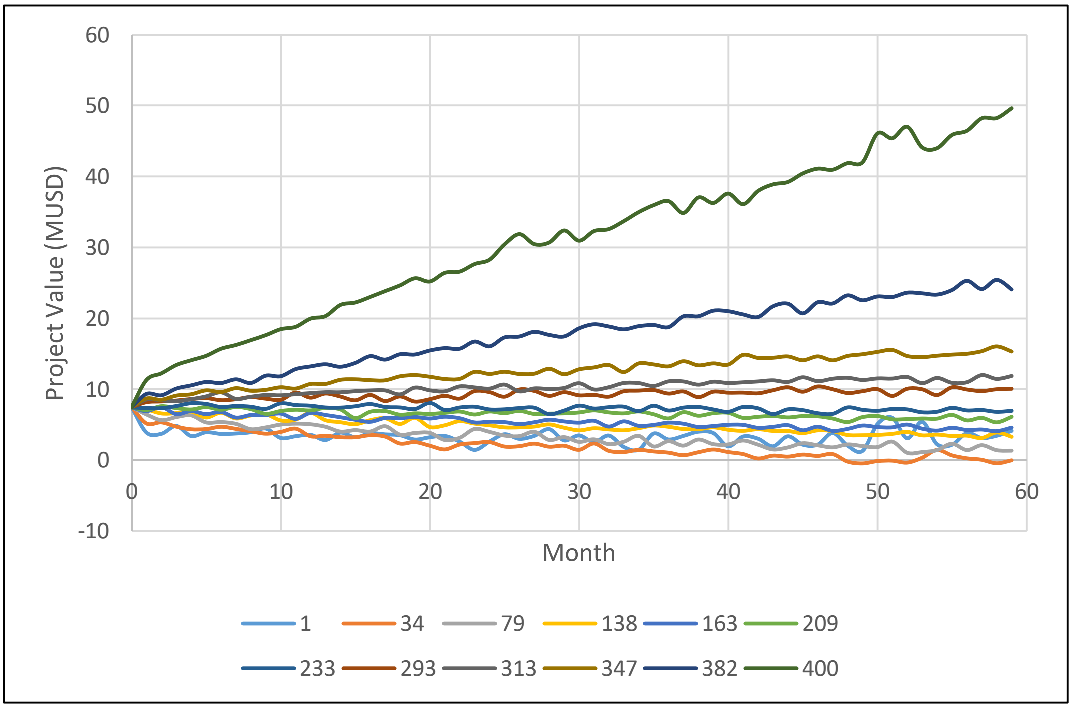

Figure 4.20 Waiting Value with Three Uncertainties Driver (Random Cells Picked)

invest and wait to invest in future time. Furthermore, the project value will be the maximum values between the exercise option computation and wait-to-operate value computations in each cell at each time.

Combination of the three uncertainties is aimed to evaluate the project with the uncertainties. The exercise and waiting values are compared at each period to decide whether to proceed with expansion or wait for more favorable economic conditions. These are computed in the backward calculation. Each cell's exercise and waiting value will always be compared to obtain the project value for each cell.

The exercise, waiting, and project value computations indicated that the project values can take any value when deciding to exercise now or in the future. These results showed the impact uncertainties effects on the mining project. However, the decision-making from the java results can be made from the frequency distribution, probability distribution, and most importantly, on the output net present value and real option value with its sensitivity analysis. A total of 100,000 paths were simulated to calculate the expansion option payoff, and 84,433 were output from the payoff frequency distribution in Figure 4.21. It showed that the largest number (16,000) of paths remained at payoff range 0, implying the computation recommended to wait for the project. However, other simulations yielded positive values with most probabilities at 7 MUSD and normally distributed to 23 MUSD.

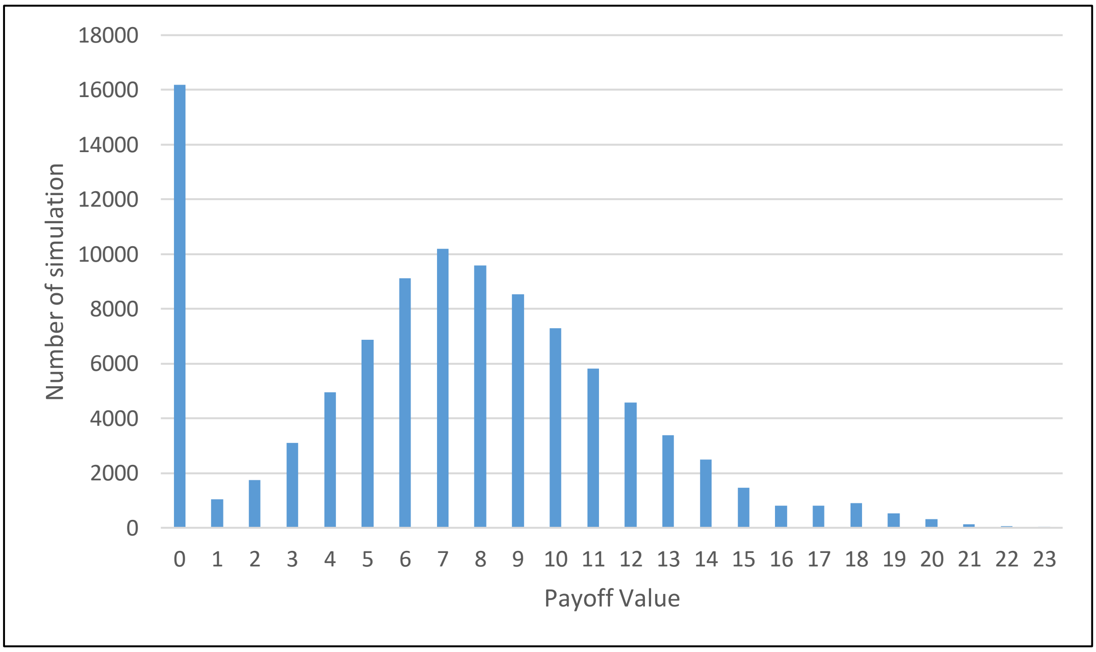

Figure 4.21 Probability Distribution

Our assumed MSSA simulation with price, grade, and cost uncertainty consideration resulted in a project real option value of 17.6 MUSD compared to 7.5 MUSD when using the traditional DCF Method. We simulated for changes in the critical parameter, which are initial price (Figure 4.22), initial total reserve (Figure 4.23), and initial total cost (Figure 4.24). With the determined uncertainty parameter, changing price and reserve gave a similar project value (Figure 4.22 and 4.23). These results showed a delaying decision recommendation with option premium reached, 135% which is far beyond our 5% threshold. Moreover, the decision recommendation shifted into abandoning the project when the project value decreased 90% due to the option value approaching zero.

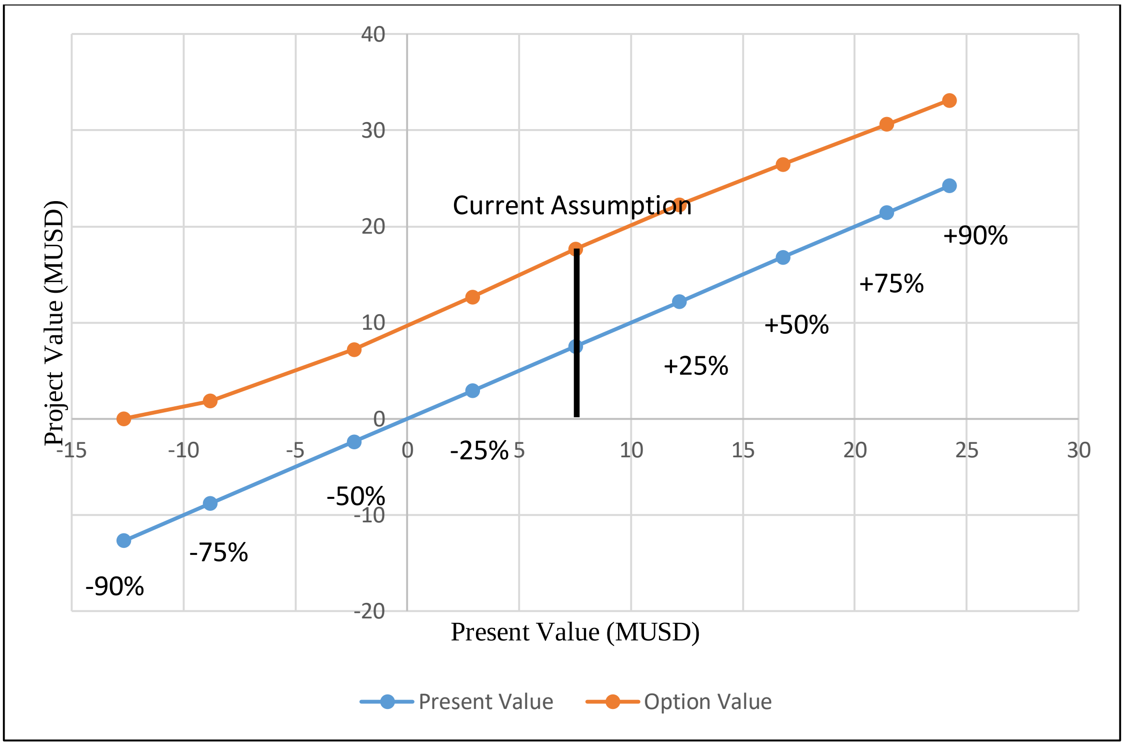

Figure 4.22 Project Value with Initial Price Changing

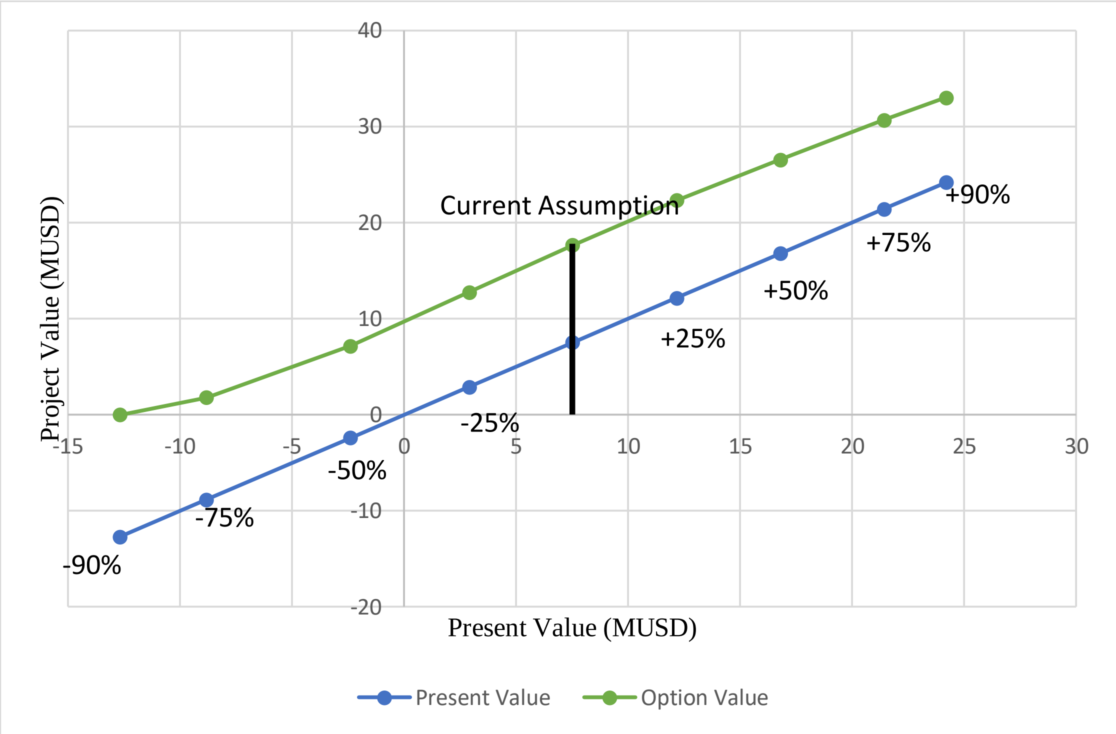

Figure 4.23 Project Value with Initial Total Reserve Changing

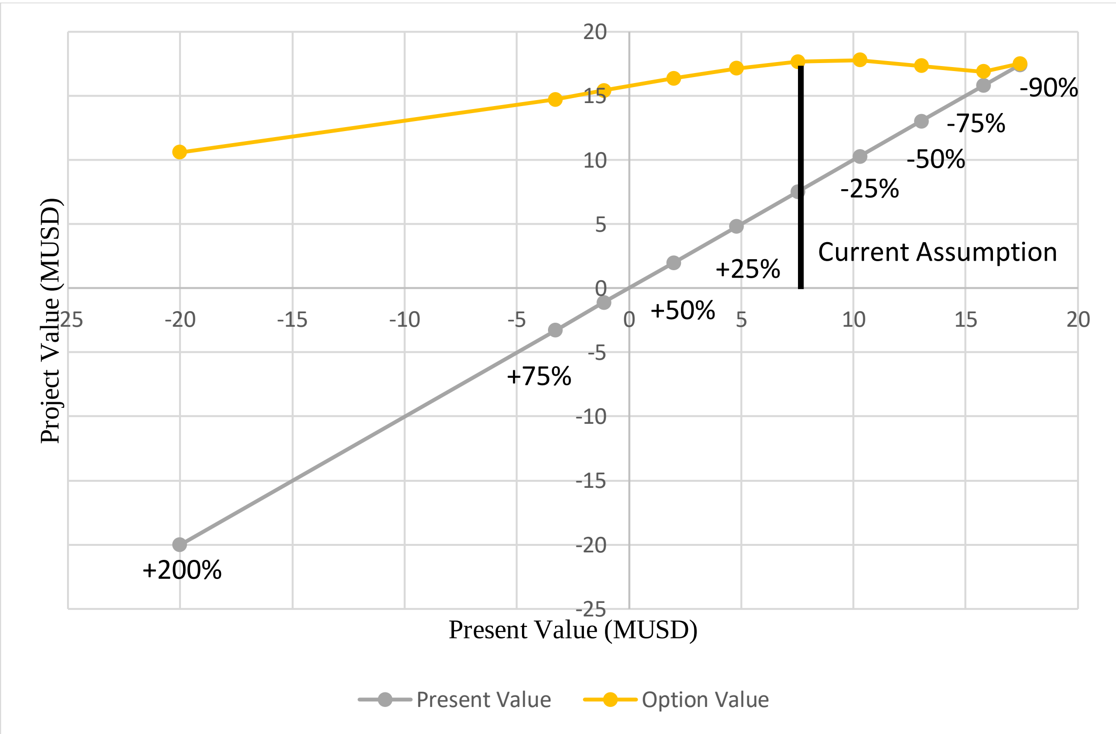

Figure 4.24 Project Value with Initial Total Cost Changing

Cost changing impacts wait-to-operate value to execution when it drops more than 75%. Otherwise, the value has remained at 17 MUSD (Figure 4.24). The option value was peaked at the project value without cost. The result explained that cost uncertainty has huge impact on option value. Therefore, we studied further about the contribution of the uncertainty. The dominant uncertainty was cost uncertainty (39%), followed by geological (9% and 16%) and price uncertainty (5%). The simulation recommended waiting for the project to be stable because of the huge option premium which is the differences between present value and option value.

Table 4.5 Present Value Regarding Price-Reserve Changing

Present Value (MUSD)		Price								
		-90%	-75%	-50%	-25%	0	25%	50%	75%	90%
Reserve	-90%	(15.00)	(14.61)	(13.97)	(13.33)	(12.68)	(12.04)	(11.40)	(10.76)	(10.37)
	-75%	(14.61)	(13.65)	(12.04)	(10.43)	(8.82)	(7.21)	(5.61)	(4.00)	(3.04)
	-50%	(13.97)	(12.04)	(8.83)	(5.61)	(2.40)	0.51	2.90	5.22	6.60
	-25%	(13.33)	(10.43)	(5.62)	(0.86)	2.90	6.38	9.85	13.31	15.41
	0	(12.68)	(8.83)	(2.40)	2.90	7.52	12.16	16.78	21.43	24.22
	25%	(12.04)	(7.21)	0.50	6.37	12.16	17.92	23.72	29.52	32.98
	50%	(11.39)	(5.61)	2.90	9.85	16.79	23.76	30.67	37.62	41.79
	75%	(10.76)	(4.00)	5.22	13.32	21.42	29.52	37.63	45.73	50.54
	90%	(10.37)	(3.04)	6.62	15.40	24.19	32.99	41.81	50.60	55.84

Table 4.6 Option Value Regarding Price-Reserve Changing

Option Value (MUSD)		Price								
		-90%	-75%	-50%	-25%	0	25%	50%	75%	90%
Reserve	-90%	-	-	-	-	-	-	-	0.19	0.53
	-75%	-	-	-	0.49	1.81	3.09	4.44	5.77	6.64
	-50%	-	-	1.82	4.41	7.15	9.97	12.68	15.17	16.70
	-25%	-	0.48	4.39	8.62	12.77	16.49	20.05	23.31	25.26
	0	-	1.82	7.18	12.64	17.66	22.24	26.46	30.62	33.09
	25%	-	3.12	10.04	16.49	22.31	27.51	32.67	37.73	40.73
	50%	-	4.42	12.69	20.01	26.55	32.69	38.72	44.71	48.41
	75%	0.21	5.77	15.26	23.32	30.68	37.74	44.85	51.87	56.21
	90%	0.54	6.60	16.72	25.27	33.02	40.80	48.43	56.23	60.96

The superiority of the real options methodology compared to the traditional method enables the manager to plan activities when uncertainty is revealed. The option value will be taken from the possibility of the improved economic parameter (Crundwell, 2008). Therefore, we made a decision map when initial price, and reserve change in the future as shown at Table 4.5, Table 4.6, Figure 4.25. The way to recommend a decision was by comparing present value and option value. With a threshold of 5% and determined price, grade and cost uncertainty parameters, MSSA methodology will always recommend holding the project. It turned to abandon recommendation when the price or reserve drops 90%. The other scenario between price and reserve value was described in Table 4.5, Table 4.6, Figure 4.25. The results of the

MSSA methodology (Table 4.5 and Figure 4.25) opposed the results of the traditional method, which recommends immediate exercising of the project. The traditional method recommended exercising the project when the present value exceeds zero. The present value becomes negative when the price or reserve drops by 50%. The overall result in both traditional and real option method is symmetrical because both affect the income value of the cash flow. Discounted Cash Flow methodology only views the projected income to determine the decision. The assumed project cash flow is sensitive to income and outcome parameters (Table 4.5). Price and reserve parameters are the income side of the project. Both parameters will be revealed when the company decides to exercise the project.

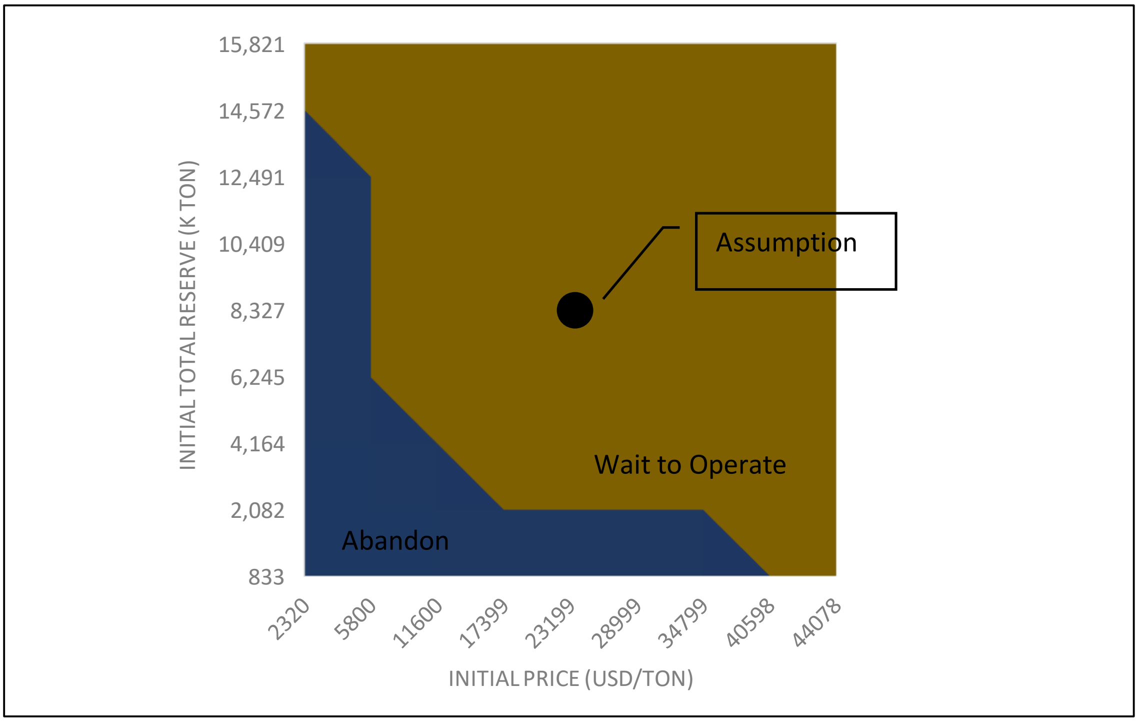

Figure 4.25 Initial Price and Total Reserve Decision Map

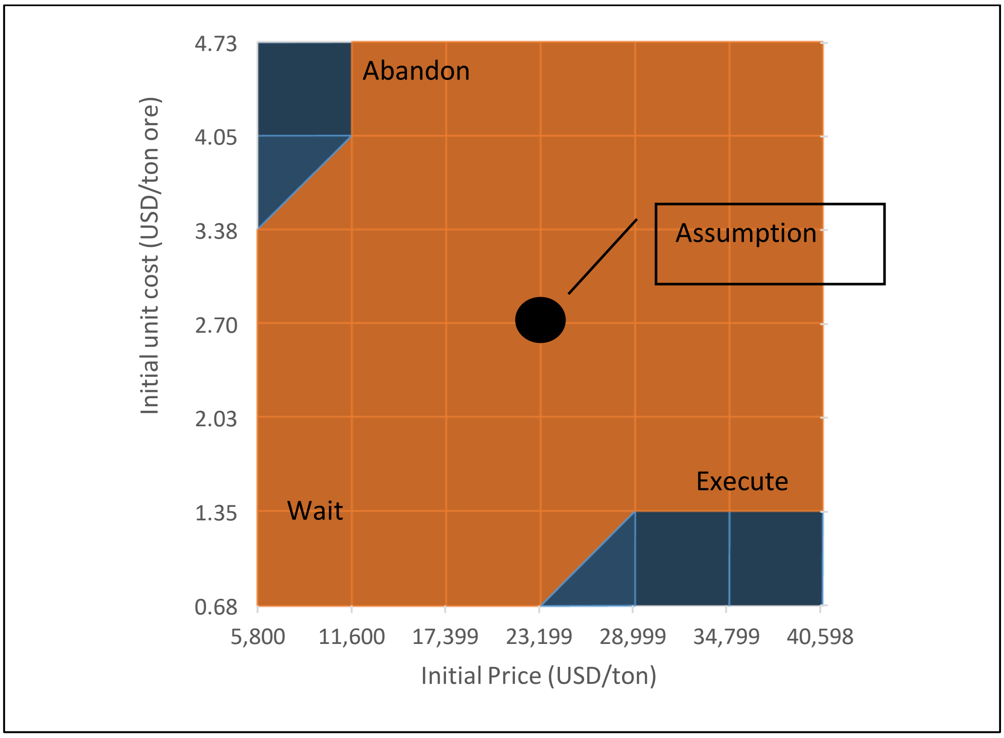

Figure 4.26 Initial Price and Unit Cost Decision Map

Figure 4.25 and Figure 4.26 showed a condition of decision recommendation considering the initial price, total reserve, and monthly cost shifting. It is divided into two areas which are waiting to operate and abandoning the project. When the mine planning is done, the initial condition was a tin price of 23,199 USD/ton and obtained expected total reserve from conditional simulation of 8.3 Mton. On the other hand, the unit cost to operate the two mining areas was 2.7 USD/ton ore. Those conditions resulted in wait-to-operate decision recommendations when considering price, grade, and cost uncertainties. The decision can shift in a certain situation as shown in the maps (Figure 4.25 and Figure 4.26).

Price and reserve uncertainties had similar properties related to the shift in present and option values. However, cost shifting had different outcome patern. Increasing cost definitely decreasing value of the project. The option premium as determinant of decision recommendation can be decreased by increasing the profit or decreasing the uncertainties driver. We have simulated further for the uncertainties driver at Figure 4.27.

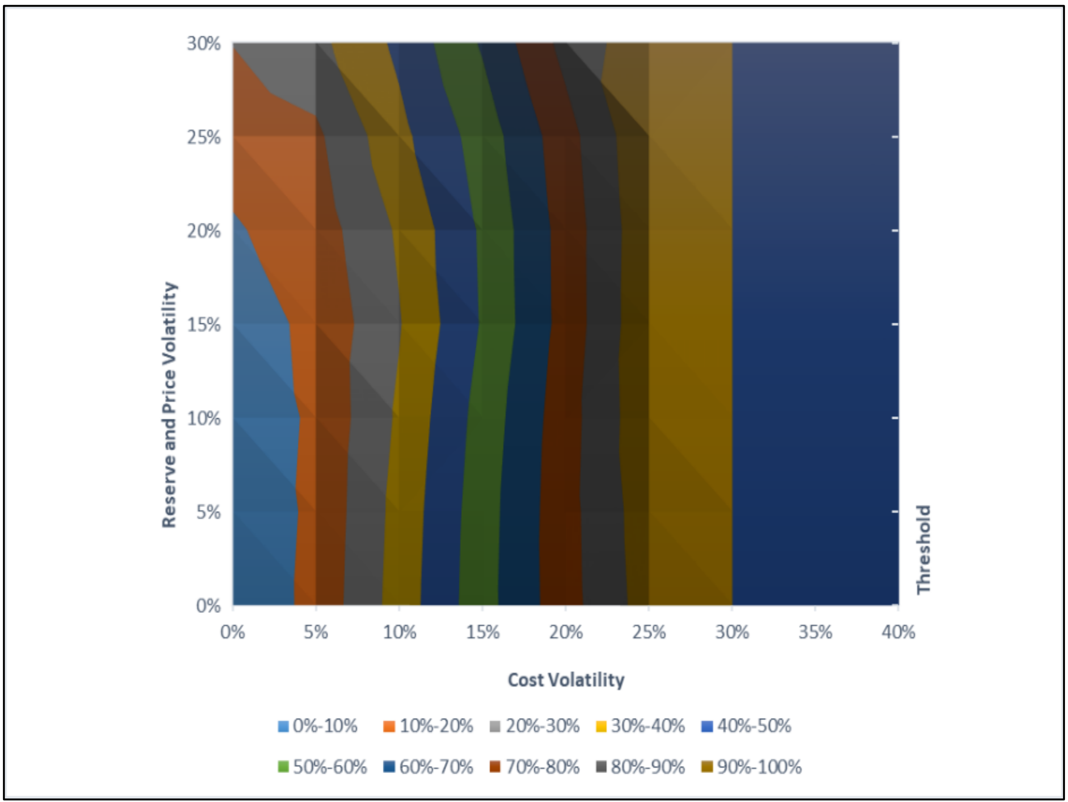

Figure 4.27 Reserve - Cost Uncertainty Scenario Analysis

Figure 4.27 showed that project uncertainties accounted for option premium in the real options methodology. Price and reserve volatility is combined regarding their similar properties of affecting income side of the project, while cost uncertainty separated to assess the result. The figure explained that cost uncertainty was the most dominant factor of the option premium compared to other uncertainties. For instance, if the company decides to have a 10% maximum risk, they should decrease the cost volatility below 5%, and so on. Otherwise, the project is delayed.

Nevertheless, the uncertainty alteration does not change the decision using a traditional methodology represented by the present value. The simulation has proven that cost uncertainty assurance is the key to project value using MSSA methodology. Additionally, the changing price and reserve are related to the present and option value shift. However, it made a pattern with the constant of option premium. Therefore, we simulated further for the no-cost uncertainty case.

4.6. Economic and Geological Uncertainty Case

Notating the distinction of cost parameters, we investigated the project value with no cost uncertainty (Figure 4.28, and Figure 4.29). The assumption is the company can control the cost to secure the profit. Our multi-stage SSA simulation with no cost uncertainty case yielded an RO value of 7.8 MUSD, compared to the corresponding net present value (NPV) of 7.5 MUSD obtained from the standard DCF method. The simulation resulted in a significant waiting value decreasing from 17.6 MUSD. The magnitude of the cost uncertainty parameter affected the decision recommendation significantly. In addition, we modeled the changes using important parameters: initial price and initial reserves (Figure 4.28, and Figure 4.29). They were treated as if their positive and negative values fluctuated. Nonetheless, the critical value of changing decision recommendation was specified when the option value approached the present value and was close to zero.

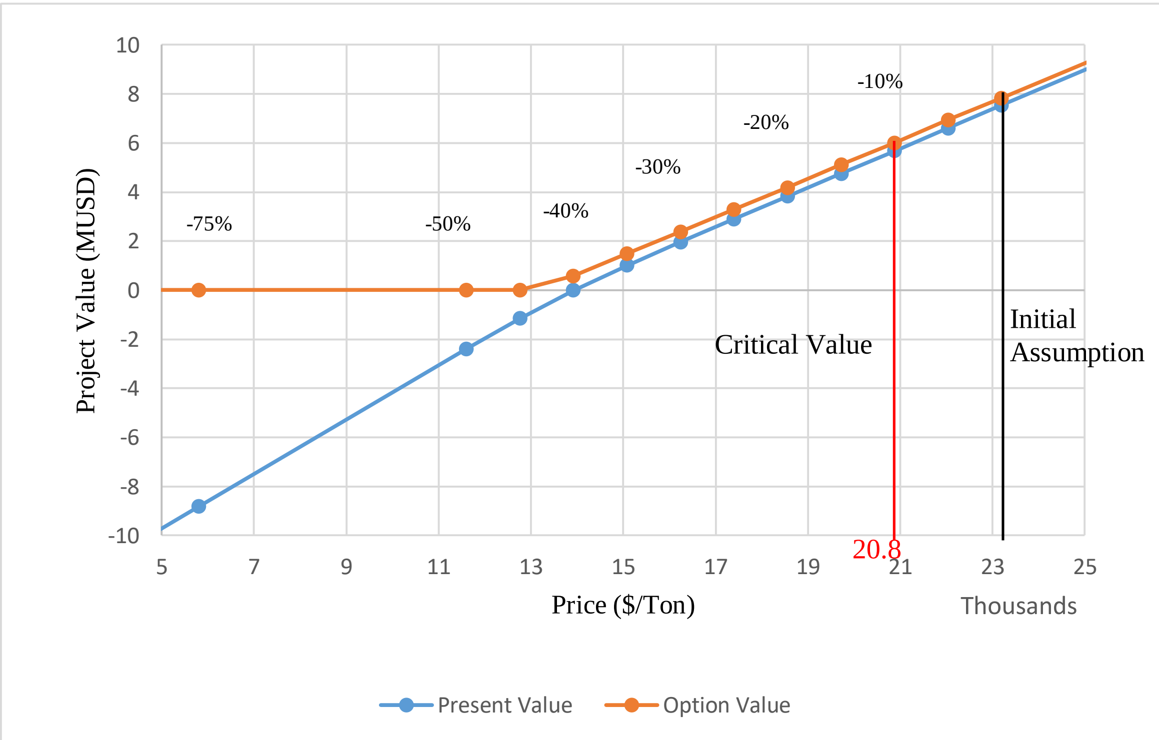

Figure 4.28 Project Value with Price Changing

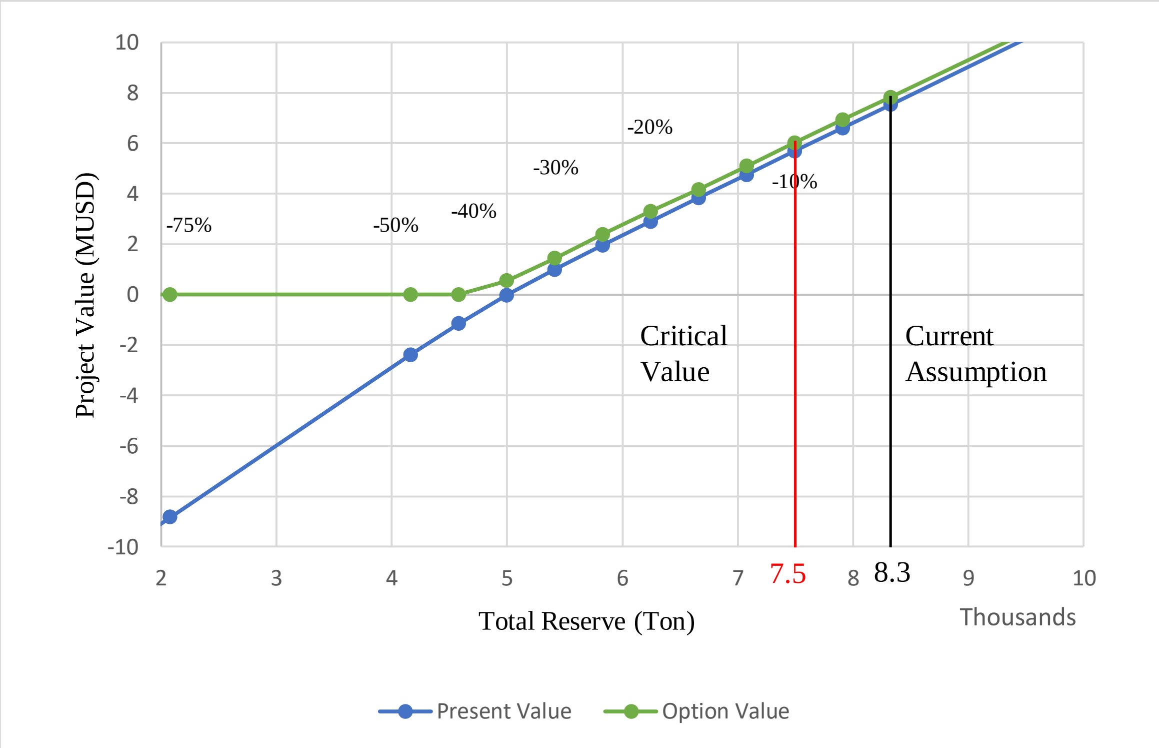

Figure 4.29 Project Value with Reserve Changing

When the price dropped by more than 10% from our assumption, the option premium value surpassed the assumed 5% threshold (Figure 4.28). Because the trend of option premium was increasing, the wait to execute decision was recommended to delay the execution of the project. When the price dropped by 45% and the RO value was almost zero, the decision was changed to "abandon the project."

The changing reserves parameter (Figure 4.29) had a similar result to the price alteration (Figure 4.28). A decrease of 10% reserve from our expected CS shifted the decision from execution to wait until the uncertainty was resolved. Moreover, when the reserves dropped by 45% from the estimated value in CS, the recommendation was changed from waiting to execute the project to abandon it as the threshold exceeded 5%.

Scenario analysis was performed to map the decision recommendation and spread from the original assumption shift in price and reserve grade. Table 4.7 and Table 4.8 represent the project's present value and option value, respectively. Recommending a decision through RO was performed by comparing the present and option values. In contrast, the traditional method produced the present value as the only decision parameter.

Table 4.7 Present Value of Reserve and Price Relative Changing

Present Value (MUSD)		Price								
		-90%	-75%	-50%	-25%	0	25%	50%	75%	90%
Reserve	-25%	(13.32)	(10.43)	(5.61)	(0.87)	2.90	6.37	9.86	13.31	15.39
	-20%	(13.20)	(10.12)	(4.97)	(0.02)	3.83	7.54	11.24	14.93	17.16
	-15%	(13.07)	(9.79)	(4.32)	0.76	4.75	8.68	12.63	16.55	18.92
	-10%	(12.94)	(9.47)	(3.68)	1.48	5.69	9.84	14.02	18.18	20.67
	-5%	(12.81)	(9.15)	(3.04)	2.20	6.62	11.00	15.41	19.78	22.42
	0	(12.68)	(8.83)	(2.40)	2.90	7.54	12.18	16.80	21.38	24.20
	5%	(12.55)	(8.51)	(1.77)	3.59	8.47	13.31	18.18	23.04	25.95
	10%	(12.43)	(8.18)	(1.16)	4.30	9.39	14.49	19.57	24.67	27.69
	15%	(12.30)	(7.86)	(0.58)	4.99	10.31	15.64	20.95	26.30	29.48
	20%	(12.17)	(7.54)	(0.02)	5.68	11.26	16.79	22.32	27.89	31.24
	25%	(12.04)	(7.22)	0.50	6.38	12.17	17.96	23.73	29.52	32.99

Table 4.8 Option Value of Reserve and Price Relative Changing

Option Value (MUSD)		Price								
		-90%	-75%	-50%	-25%	0	25%	50%	75%	90%
Reserve	-25%	-	-	-	-	3.29	6.67	10.14	13.55	15.60
	-20%	-	-	-	0.52	4.20	7.85	11.47	15.17	17.35
	-15%	-	-	-	1.25	5.10	8.96	12.87	16.77	19.10
	-10%	-	-	-	1.95	6.01	10.11	14.25	18.39	20.90
	-5%	-	-	-	2.60	6.92	11.26	15.60	20.03	22.59
	0	-	-	-	3.29	7.82	12.39	16.97	21.57	24.35
	5%	-	-	-	3.95	8.72	13.53	18.40	23.23	26.15
	10%	-	-	-	4.67	9.66	14.70	19.78	24.89	27.90
	15%	-	-	0.05	5.35	10.52	15.83	21.13	26.47	29.65
	20%	-	-	0.62	6.03	11.51	17.01	22.51	28.11	31.42
	25%	-	-	1.03	6.70	12.39	18.09	23.88	29.70	33.18

The colored blocks in Table 4.7 represent the decision recommendation through the traditional DCF method. A positive present value would be reached if the price exceeded 12,000 dollars per ton ore or drops 50% from our initial assumption. With option premium due to the parameter's wait-to-operate value resulting from uncertainty, the RO methodology and the SSA approach improved the decision to neither execute (green color) nor reject (red color) the mine project. Instead, it provided waiting recommendations for specific cases, as shown by the yellow color in Table 4.8 and Figure 4.30.

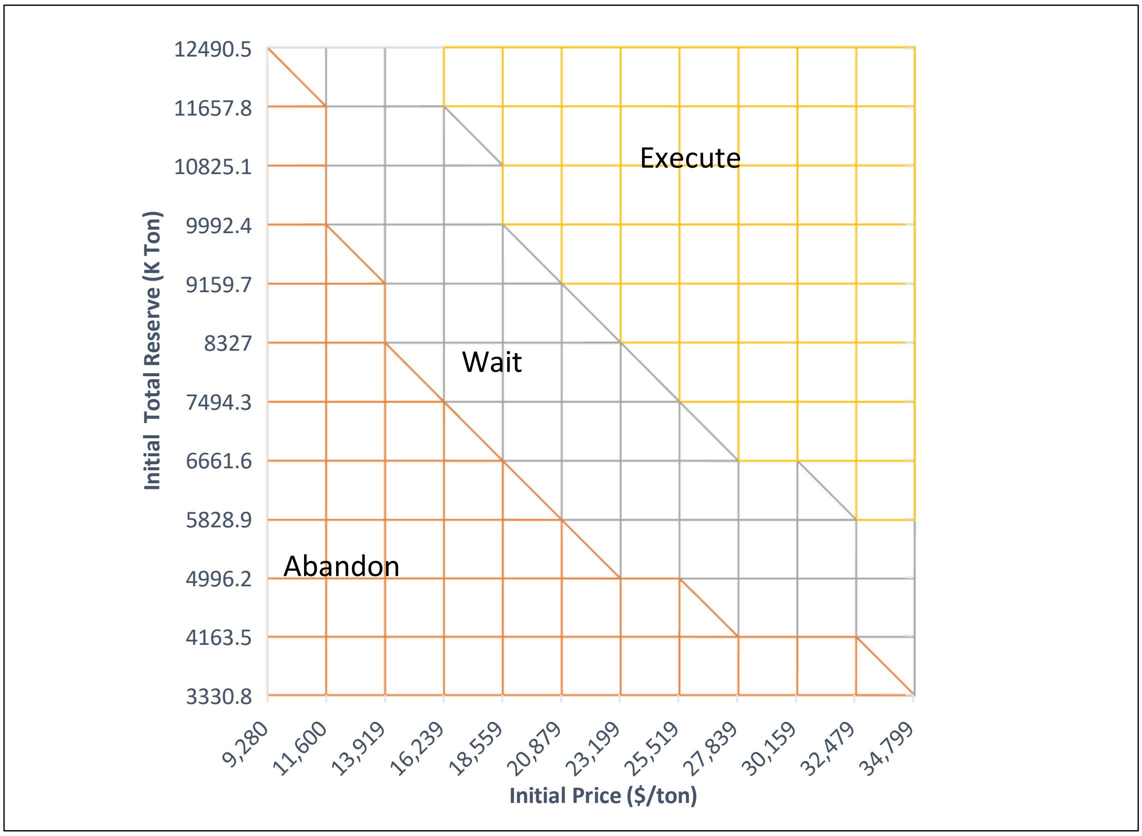

Figure 4.30 Decision Initial Price-Reserve Map

The DCF methodology only views projected income to determine the decision. However, the assumed project cash flow was sensitive to income and outcome parameters. The calculation of option premium resulted a risk that should be paid by company if they decided to continue the project. Therefore, the risk apatite of a manager will be determinant of execution decision. Theoretically, higher risk should give higher return and vice versa. In this research, 5% threshold was applied to recommend a decision. Shifting initial price and total reserve as uncertainty driver was assessed. Current assumption stood at the borderline of execution and wait to operate decision. The recommended decision can be shifted with changing price, reserve or increasing the risk threshold.

Uncertainties were not accounted for in the traditional method; therefore, they did not affect the present value and remained constant at 7.5 MUSD either using manual-based DCF calculations or SSA through the JAVA program language. These calculations summarized the cash flow model with the additional time value of money. Figure 4.31 displays the option value alteration where the uncertainty was changed in increments of 5%. The value of waiting implied that the project had to be delayed when the option value was far from the calculated present value. A shift in the price uncertainty had an almost symmetrical effect with the reserves uncertainty to the option value. When the parameter was altered below the critical value of 8.3 MUSD, the decision recommendation was changed from execution to wait-to-operate or to abandon the project. The price and reserves parameters presented the income side of the project; therefore, both parameters were incorporated when the company decided to exercise the project. The two uncertainties had similar properties, implying future decisions based on future conditions.

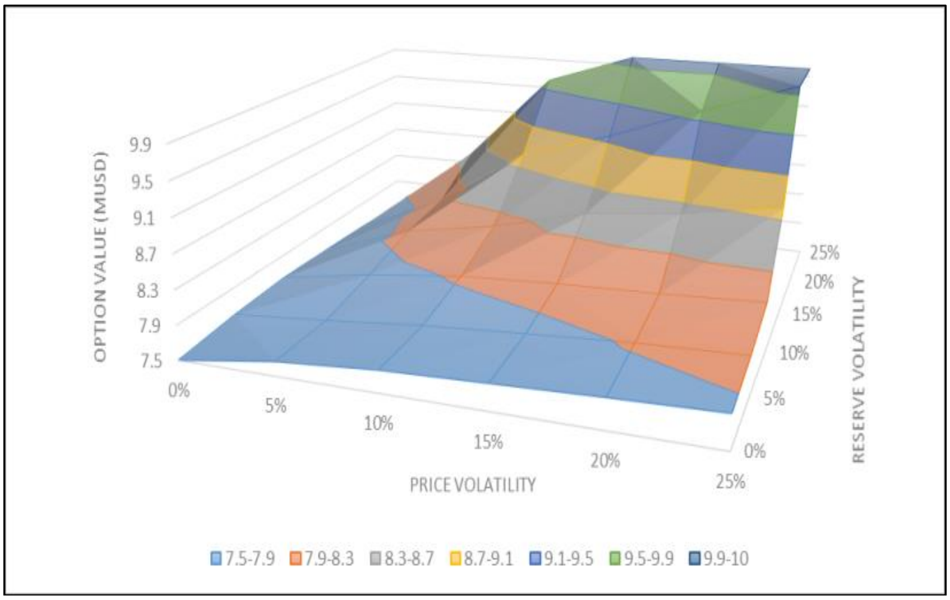

Figure 4.31 Real Option (RO) Value with Price and Reserve Uncertainties Alteration

A mining company does not have control over price and reserve. Price will always fluctuate in the future, which depends on the supply and demand of the commodity. On the other hand, the reserve is a spatial base parameter. It will be known when mining the reserve. Otherwise, the company should increase the exploration drilling to assure the reserve. The spacing between drill holes determines the volatility. It can be additional data to confirm the reserve amount and decrease the geological uncertainty. However, the geological uncertainty will never be eliminated caused by the nugget effect. The term "nugget" is borrowed from geostatistics, referring to the unexpected nugget of minerals founding a mining process (Yin J et al., 2011). Cressie (1993) concluded that the nugget effect in geostatistics is caused by two factors: micro-scale variation and measurement error.

The critical value was calculated to secure the project profitability. Dixit and Pindyck emphasized that this critical value is reached when the waiting value is identical to or less than the executing value (Dixit & Pindyck, 1993). However, the uncertainties created distance between the two values. Mun stated that the company's behavior determined the risk of a project (Mun, 2006). Therefore, the critical value may vary among decision-makers. Although both methods recommended immediate execution of the project, SSA improved the results of the traditional method. The traditional method recommended executing the project when the present value exceeded zero. This improvement caused the RO methodology to recommend the delay option under several other conditions.

This study further demonstrates the application of RO in the mining industry. It successfully combines the advancement of geostatistics in terms of conditional simulation and real option valuation through the SSA approach. The current limitation concerning real options, which include mathematical complexity and applicability (Haque et al., 2014), is solved using our methodology. Monte Carlo simulation solves the mathematical problem while incorporating grade uncertainty to determine the application, particularly mining. Capturing additional uncertainty increases the future applicability of the RO. Each modifying factor in the conversion resource to be reserved and from geological confidence (The JORC Code 2012 Edition, 2012) is supposed to be modeled further to obtain proper project value and ultimately guarantee a successful project with calculated risk.

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Chapter 5 Conclusions

In summary, we had constructed a valuation method using the developed MSSA methodology. The method is adapted from well-known financial theory, namely option analysis, which have similarities in analyzing risk in the real business. An option is a financial derivative whose value depends on the price of a stock. Option holders have rights but no obligation to execute their option in the maturity date. These alternatives are quantified by option calculation to justify option value. In this thesis, we applied it at a mining project in Indonesia, particularly production stage of underwater tin mine planning. The objective of the valuation is giving recommendation to mine company whether execute the project, delay or abandon it when the condition is not favorable. We addressed the risk of price, reserve, and cost which represented economical, geological and technical uncertainties. Our established valuation approach addressed limitations in RO applications, particularly concerning assessing several sources of uncertainty in a multi-stage short-term mining project. The application exhibited practical simplicity as well as applicability. We enhanced previous studies by employing the adaptation of spatial uncertainty through conditional simulation (CS) and the construct path generation algorithm to incorporate the three uncertainties.

The mining project had been planned using GEOVIA Surpac and Whittle Mining Software based on real tin exploration data. The assumed price was at January 2020 while cost is calculated at the average historical data in 5 years with similar mining method and equipment when the project was planned. The obtained present value for twos mining areas was 7.5 million USD. Uncertainty in mining could be modeled using the real option and our result was 17.6 million USD with price, grade and cost uncertainties driver. Thus, the MSSA approach in real option methodology evaluated multi-uncertainty and multi-stage mining (rainbow option). Our developed algorithm could produce real option value using JAVA Programming as a simulation tool.

Generally, a mining company could not control the price and reserve uncertainties. We used the Geometric Brownian model to account for price and reserve uncertainties. While for cost uncertainty, the Mean Reverting model gave a better result. Price would be revealed when the company decided to mine. At the same time, the reserve will be proven by the production. The increasing of exploration drill data theoretically could decrease the uncertainty. However, a huge exploration expenditure would be the consequence.

Logarithmic natural growth analysis based on historical data resulted in the price has an average monthly growth rate (α) of 0.74%. However, to be applied at our evaluation method, we assumed the price growth would follow the stochastic differential equation model, particularly the one-factor constant convenience yield model for risk-neutral prices. Assumed parameters were risk-free rate of 0.25%, convenience yield 0.17% and calculated price volatility of 4.7%.

This research started to utilize the CS method to quantify the reserve uncertainty in real options valuation. CS is a geostatistical tool that can generate punctually or block 'realizations' of mineral grades. Each realization is intended to honor the histogram and semivariogram of the true grade distribution and honor known data points. It is a technique to measure spatial uncertainty based on spatial Monte Carlo Analysis (MCS). CS reproduces the properties of an ore body and quantifies the variability, which we call "spatial uncertainty." We incorporated the advanced geostatistical application as well as advanced economic valuation methodology.

We measure the technical uncertainty represented by cost volatility. The cost consisted of mining, processing and metallurgical cost. Unit cost of production model construction was based on historical cost given by the company with a similar location and production technique. Similar ADF methodology was applied to the summarized historical unit cost data. The unit root test concluded that the cost data is stationer. Therefore, the construction of the technical uncertainty model was used the Mean Reverting (MR) model. We used the simplest mean-reverting process-also known as the Ornstein-Uhlenbeck process.

A mining company does not have control over price and reserve. Price will always fluctuate in the future, which depends on the supply and demand of the commodity. On the other hand, the reserve is a spatial base parameter. It will be known when mining the reserve. We simulated the combination relative shifting and volatility of price, reserve and cost. The real option result opposed traditional Discounted Cash Flow in several condition particularly in the waiting decision condition. In real option method, option premium is the decision determinant. Our 5% threshold assumption recommended to execute the project when the company can eliminate cost volatility. The condition resulted option premium 3.8% which dropped from 135% when cost uncertainty is existing. The simulation revealed that cost uncertainty of 39% was the dominant factor of project valuation using real options methodology comparing to price uncertainty (4.7%) and grade uncertainty (9% and 16%). The company should ensure cost planning before executing the project to make the project profitable.

There are many uncertainties in the mining industry. The most significant uncertainties are economical, geological, and technical, represented by price, grade, and cost. The traditional method could not explain those uncertainties, and the previous real options method did not incorporate the uncertainties. This research is a pilot project for combining uncertainties simultaneously through the MSSA approach. Practically, a mining company can execute at calculated risk or wait to resolve uncertainty or abandon the project to save from loss. Additionally, we cannot perform standard real options approaches Black-Scholes and lattice method to evaluate multi uncertainties. These methodologies resulted only in price risk measurement by project. They cannot assess an ongoing project and multi uncertainties. In contrast, the MSSA could capture more permutation of the cash flow parameters. Nonetheless, the MSSA needed adaptation to model each uncertainty faced by the mining company and high computation power to simulate.

We simulated further for changing parameter of uncertainty. Price, grade and cost is assumed as control variable to get critical value. Moreover, those uncertainty is shifted to obtain profitable project. The uncertainty alteration does not change the decision using a traditional methodology represented by the present value. The simulation has proven that cost uncertainty assurance is the key to project value using MSSA methodology. Additionally, the changing price and reserve are related to the present and option value shift identically. However, it made a pattern with the constant of option premium.

The current limitation for RO, including mathematical complexity and applicability, is solved using our methodology. SSA simulation solves the mathematical problem while incorporating grade uncertainty in the simulation to determine the project value and risk. However, the SSA methodology needs optimization on number of simulation and modeling methods. Balancing calculation time and consistency of result is the key of method limitation. Furthermore, comparing to Black-Scholes and Binomial Lattice method, only SSA can be performed on the case study. Adding capital assumption and eliminating grade uncertainty, they have similar result with difference less than 1%. Nonetheless, capturing additional uncertainty increases the future applicability of the RO. Each modifying factor in the conversion resource to be reserved and from geological confidence is supposed to be modeled further to obtain proper project value and ultimately guarantee a successful project with calculated risk.

This study presented a project that combined uncertainties and multi projects in mining ventures. We considered pricing and geological factors in two mining locations, whereas other modifying factors were assumed to be stable. Owing to its ability to capture real-world parameters, the utilization of the proposed RO methodology in the mining industry is expected to increase in future. In terms of future development, the valuation method could be further enhanced to combine additional uncertainties, project valuation, and stages of investment. The outcome parameter in cash flow in terms of mining expenditure may present an additional application of the proposed methodology. The method can be developed further to combine more uncertainties, project valuation, and more stages of investment. The utilization of real options methodology in the mining industry is expected to increase by capturing the real uncertainty as stated at modifying factor to convert resource to be reserve. Finally, uncertainties could be overcome by management's flexibility to either hold or abandon the project.

APPENDIX

A. Price Modelling

Date	Price Cash ($/tonne)	Log Price Change (Ui)	ΣUi	Ui2	ΣUi2	n	s	Δt	σ	Notes
1/1/2016	14,906.00	0.0693	0.4423	0.0048	0.1406	61	0.047448	1	4.74%	Monthly volatility
2/1/2016	15,976.00	0.0461		0.0021						
3/1/2016	16,729.00	0.0312		0.0010						
4/1/2016	17,260.00	-0.0558		0.0031						
5/1/2016	16,324.00	0.0450		0.0020						
6/1/2016	17,075.00	0.0438		0.0019						
7/1/2016	17,840.00	0.0568		0.0032						
8/1/2016	18,882.00	0.0657		0.0043						
9/1/2016	20,164.00	0.0351		0.0012						
10/1/2016	20,885.00	0.0206		0.0004						
11/1/2016	21,320.00	-0.0054		0.0000						
12/1/2016	21,205.00	-0.0694		0.0048						
1/1/2017	19,783.00	-0.0297		0.0009						
2/1/2017	19,205.00	0.0520		0.0027						
3/1/2017	20,230.00	-0.0119		0.0001						
4/1/2017	19,990.00	0.0220		0.0005						
5/1/2017	20,435.00	-0.0101		0.0001						
6/1/2017	20,230.00	0.0275		0.0008						
7/1/2017	20,795.00	0.0037		0.0000						
8/1/2017	20,873.00	-0.0013		0.0000						
9/1/2017	20,845.00	-0.0664		0.0044						
10/1/2017	19,505.00	0.0094		0.0001						
11/1/2017	19,690.00	0.0204		0.0004						
12/1/2017	20,096.00	0.0839		0.0070						

Date	Price Cash ($/tonne)	Log Price Change (Ui)	ΣUi	Ui2	ΣUi2	n	s	Δt	σ	Notes
1/1/2018	21,855.00	-0.0099		0.0001						
2/1/2018	21,640.00	-0.0217		0.0005						
3/1/2018	21,175.00	0.0066		0.0000						
4/1/2018	21,315.00	-0.0327		0.0011						
5/1/2018	20,630.00	-0.0396		0.0016						
6/1/2018	19,830.00	0.0172		0.0003						
7/1/2018	20,175.00	-0.0572		0.0033						
8/1/2018	19,052.50	-0.0102		0.0001						
9/1/2018	18,860.00	0.0146		0.0002						
10/1/2018	19,138.00	-0.0394		0.0016						
11/1/2018	18,398.00	0.0592		0.0035						
12/1/2018	19,520.00	0.0697		0.0049						
1/1/2019	20,930.00	0.0383		0.0015						
2/1/2019	21,747.00	-0.0139		0.0002						
3/1/2019	21,447.00	-0.0829		0.0069						
4/1/2019	19,741.00	-0.0475		0.0023						
5/1/2019	18,825.00	0.0004		0.0000						
6/1/2019	18,833.00	-0.0840		0.0071						
7/1/2019	17,315.00	-0.0573		0.0033						
8/1/2019	16,350.00	-0.0273		0.0007						
9/1/2019	15,910.00	0.0358		0.0013						
10/1/2019	16,490.00	0.0008		0.0000						
11/1/2019	16,504.00	0.0400		0.0016						
12/1/2019	17,178.00	-0.0448		0.0020						
1/1/2020	16,425.00	-0.0097		0.0001						
2/1/2020	16,267.00	-0.1035		0.0107						
3/1/2020	14,667.00	0.0406		0.0016						
4/1/2020	15,274.00	0.0149		0.0002						
5/1/2020	15,502.85	0.0815		0.0066						
6/1/2020	16,819.00	0.0628		0.0039						

Date	Price Cash ($/tonne)	Log Price Change (Ui)	ΣUi	Ui2	ΣUi2	n	s	Δt	σ	Notes
7/1/2020	17,909.00	-0.0202		0.0004						
8/1/2020	17,550.00	-0.0058		0.0000						
9/1/2020	17,448.75	0.0157		0.0002						
10/1/2020	17,724.00	0.0505		0.0025						
11/1/2020	18,642.00	0.0972		0.0094						
12/1/2020	20,544.50	0.1215		0.0148						
1/1/2021	23,199.00			0.0000						
Average	18,870.06	0.00737242								
Notes		Drift rate				Data Amount	Standard Deviation	Period	Variance	

B. Resource Summary

South Resource Summary

Grade Range (kg/ ton)	Minimum Grade (kg/ ton)	Maximum Grade (kg/ ton)	Expected Grade (kg/ ton)	Std dev (kg/ ton)	Tonnes Cummulative	Volume (m3)	Tonnes
0.0 -> 0.1	0.01	0.138	0.03	0.01	1,205,250	128,375	385,125
0.1 -> 0.2	0.01	0.627	0.143	0.08	820,125	35,000	105,000
0.2 -> 0.3	0.01	1.084	0.253	0.16	715,125	8,000	24,000
0.3 -> 0.4	0.06	1.825	0.357	0.19	691,125	11,625	34,875
0.4 -> 0.5	0.09	1.144	0.449	0.17	656,250	14,625	43,875
0.5 -> 0.6	0.15	1.239	0.555	0.17	612,375	29,000	87,000
0.6 -> 0.7	0.22	1.436	0.645	0.20	525,375	33,625	100,875
0.7 -> 0.8	0.23	1.597	0.749	0.24	424,500	18,625	55,875
0.8 -> 0.9	0.23	2.244	0.846	0.34	368,625	13,750	41,250
0.9 -> 1.0	0.23	2.614	0.949	0.36	327,375	10,750	32,250
1.0 -> 1.1	0.28	2.518	1.052	0.34	295,125	11,750	35,250
1.1 -> 1.2	0.36	2.423	1.155	0.34	259,875	13,375	40,125
1.2 -> 1.3	0.36	2.605	1.246	0.37	219,750	13,875	41,625
1.3 -> 1.4	0.26	2.845	1.351	0.39	178,125	15,625	46,875
1.4 -> 1.5	0.48	2.952	1.444	0.42	131,250	12,625	37,875
1.5 -> 1.6	0.35	3.15	1.549	0.45	93,375	9,625	28,875
1.6 -> 1.7	0.39	3.642	1.646	0.50	64,500	9,000	27,000
1.7 -> 1.8	0.20	4.308	1.747	0.67	37,500	4,500	13,500
1.8 -> 1.9	0.11	5.757	1.84	0.86	24,000	4,625	13,875
1.9 -> 2.0	0.04	5.908	1.947	1.11	10,125	2,875	8,625
2.0 -> 2.1	0.02	7	2.031	1.28	1,500	500	1,500
Grand Total	0.328	0.905	0.578	0.09		401,750	1,205,250

Mid Resource Summary

Grade Range (kg/ ton)	Minimum Grade (kg/ ton)	Maximum Grade (kg/ ton)	Expected Grade (kg/ ton)	Std dev (kg/ ton)	Tonnes Cummulative	Volume (m3)	Tonnes
0.0 -> 0.1	0.02	0.08	0.04	0.01	11,503,500	964,500	2,893,500
0.1 -> 0.2	0.06	0.28	0.15	0.03	8,610,000	484,500	1,453,500
0.2 -> 0.3	0.10	0.46	0.25	0.05	7,156,500	332,000	996,000
0.3 -> 0.4	0.18	0.60	0.35	0.07	6,160,500	308,500	925,500
0.4 -> 0.5	0.23	0.71	0.45	0.08	5,235,000	258,000	774,000
0.5 -> 0.6	0.30	0.88	0.55	0.10	4,461,000	227,500	682,500
0.6 -> 0.7	0.33	1.10	0.65	0.12	3,778,500	151,000	453,000
0.7 -> 0.8	0.38	1.30	0.75	0.13	3,325,500	160,000	480,000
0.8 -> 0.9	0.49	1.38	0.85	0.14	2,845,500	169,500	508,500
0.9 -> 1.0	0.54	1.51	0.95	0.15	2,337,000	177,000	531,000
1.0 -> 1.1	0.53	1.73	1.05	0.18	1,806,000	138,500	415,500
1.1 -> 1.2	0.57	1.84	1.15	0.19	1,390,500	105,000	315,000
1.2 -> 1.3	0.61	1.92	1.25	0.22	1,075,500	90,000	270,000
1.3 -> 1.4	0.76	2.10	1.35	0.24	805,500	97,000	291,000
1.4 -> 1.5	0.72	2.39	1.45	0.27	514,500	84,500	253,500
1.5 -> 1.6	0.59	2.71	1.54	0.35	261,000	44,500	133,500
1.6 -> 1.7	0.33	3.69	1.64	0.53	127,500	16,500	49,500
1.7 -> 1.8	0.13	4.14	1.74	0.66	78,000	8,500	25,500
1.8 -> 1.9	0.07	5.61	1.85	0.91	52,500	6,000	18,000
1.9 -> 2.0	0.03	5.24	1.96	1.03	34,500	4,500	13,500
2.0 -> 2.1	0.02	6.42	2.04	1.18	21,000	3,500	10,500
2.1 -> 2.2	0.00	7.00	2.12	1.88	10,500	1,000	3,000
2.2 -> 2.3	-	7.00	2.27	2.71	7,500	500	1,500
2.4 -> 2.5	-	7.00	2.43	2.77	6,000	500	1,500

Grade Range (kg/ ton)	Minimum Grade (kg/ ton)	Maximum Grade (kg/ ton)	Expected Grade (kg/ ton)	Std dev (kg/ ton)	Tonnes Cummulative	Volume (m3)	Tonnes
2.6 -> 2.7	-	7.00	2.66	2.79	4,500	500	1,500
2.7 -> 2.8	-	7.00	2.74	2.89	3,000	500	1,500
3.0 -> 3.1	-	7.00	3.08	2.84	1,500	500	1,500
Grand Total	0.37	0.60	0.48	0.04		3,834,500	11,503,500

C. Reserve Summary

South Reserve Summary

Grade Range (kg/ ton)	Minimum Grade (kg/ ton)	Maximum Grade (kg/ ton)	Expected Grade (kg/ ton)	Std dev (kg/ ton)	Tonnes Cummulative	Volume (m3)	Tonnes
0.0 -> 0.1	0.01	0.14	0.03	0.01	1,123,125	128,375	385,125
0.1 -> 0.2	0.01	0.63	0.14	0.08	738,000	35,000	105,000
0.2 -> 0.3	0.01	1.06	0.25	0.16	633,000	7,875	23,625
0.3 -> 0.4	0.06	1.83	0.36	0.19	609,375	11,625	34,875
0.4 -> 0.5	0.09	1.16	0.45	0.18	574,500	14,125	42,375
0.5 -> 0.6	0.13	1.27	0.55	0.18	532,125	25,375	76,125
0.6 -> 0.7	0.18	1.69	0.65	0.21	456,000	24,250	72,750
0.7 -> 0.8	0.20	1.82	0.75	0.26	383,250	13,375	40,125
0.8 -> 0.9	0.20	2.30	0.84	0.35	343,125	10,500	31,500
0.9 -> 1.0	0.22	2.63	0.95	0.37	311,625	10,375	31,125
1.0 -> 1.1	0.26	2.54	1.05	0.34	280,500	11,625	34,875
1.1 -> 1.2	0.36	2.42	1.16	0.34	245,625	13,375	40,125
1.2 -> 1.3	0.36	2.61	1.25	0.37	205,500	13,875	41,625
1.3 -> 1.4	0.26	2.85	1.35	0.39	163,875	15,625	46,875
1.4 -> 1.5	0.48	2.94	1.44	0.42	117,000	11,875	35,625

Grade Range (kg/ ton)	Minimum Grade (kg/ ton)	Maximum Grade (kg/ ton)	Expected Grade (kg/ ton)	Std dev (kg/ ton)	Tonnes Cummulative	Volume (m3)	Tonnes
1.5 -> 1.6	0.37	3.47	1.55	0.49	81,375	8,250	24,750
1.6 -> 1.7	0.27	3.75	1.65	0.53	56,625	7,875	23,625
1.7 -> 1.8	0.21	4.27	1.75	0.68	33,000	4,125	12,375
1.8 -> 1.9	0.11	6.20	1.84	0.90	20,625	4,250	12,750
1.9 -> 2.0	0.04	6.00	1.95	1.17	7,875	2,500	7,500
2.0 -> 2.1	-	7.00	2.03	2.50	375	125	375
Grand Total	0.31	0.88	0.56	0.09		374,375	1,123,125

Mid Reserve Summary

Grade Range (kg/ ton)	Minimum Grade (kg/ ton)	Maximum Grade (kg/ ton)	Expected Grade (kg/ ton)	Std dev (kg/ ton)	Volume (m3)	Tonnes	Tonnes Cummulative
0.0 -> 0.1	0.01	0.10	0.05	0.02	472,500	1,417,500	7,204,500
0.1 -> 0.2	0.05	0.37	0.14	0.05	220,500	661,500	5,787,000
0.2 -> 0.3	0.09	0.54	0.25	0.07	184,500	553,500	5,125,500
0.3 -> 0.4	0.14	0.64	0.35	0.08	193,000	579,000	4,572,000
0.4 -> 0.5	0.18	0.83	0.45	0.11	141,500	424,500	3,993,000
0.5 -> 0.6	0.25	1.03	0.55	0.12	149,000	447,000	3,568,500
0.6 -> 0.7	0.22	1.17	0.65	0.15	89,500	268,500	3,121,500
0.7 -> 0.8	0.34	1.37	0.75	0.15	116,500	349,500	2,853,000
0.8 -> 0.9	0.47	1.45	0.86	0.15	131,500	394,500	2,503,500
0.9 -> 1.0	0.52	1.56	0.95	0.16	156,500	469,500	2,109,000
1.0 -> 1.1	0.54	1.73	1.05	0.19	125,500	376,500	1,639,500
1.1 -> 1.2	0.59	1.93	1.15	0.21	92,000	276,000	1,263,000
1.2 -> 1.3	0.61	1.95	1.25	0.23	85,000	255,000	987,000
1.3 -> 1.4	0.67	2.16	1.35	0.25	87,000	261,000	732,000

Grade Range (kg/ ton)	Minimum Grade (kg/ ton)	Maximum Grade (kg/ ton)	Expected Grade (kg/ ton)	Std dev (kg/ ton)	Volume (m3)	Tonnes	Tonnes Cummulative
1.4 -> 1.5	0.69	2.34	1.45	0.27	75,500	226,500	471,000
1.5 -> 1.6	0.59	2.74	1.54	0.35	41,000	123,000	244,500
1.6 -> 1.7	0.34	3.67	1.64	0.54	16,000	48,000	121,500
1.7 -> 1.8	0.09	4.23	1.74	0.71	7,500	22,500	73,500
1.8 -> 1.9	0.07	5.61	1.85	0.91	6,000	18,000	51,000
1.9 -> 2.0	0.03	5.24	1.96	1.03	4,500	13,500	33,000
2.0 -> 2.1	0.02	6.58	2.04	1.28	3,000	9,000	19,500
2.1 -> 2.2	0.00	7.00	2.12	1.88	1,000	3,000	10,500
2.2 -> 2.3	-	7.00	2.27	2.71	500	1,500	7,500
2.4 -> 2.5	-	7.00	2.43	2.77	500	1,500	6,000
2.6 -> 2.7	-	7.00	2.66	2.79	500	1,500	4,500
2.7 -> 2.8	-	7.00	2.74	2.89	500	1,500	3,000
3.0 -> 3.1	-	7.00	3.08	2.84	500	1,500	1,500
Grand Total	0.44	0.77		0.05	2,401,500	7,204,500	

D. Cost Calculation

Operating Cost

Unit	Year	2015	2016	2017	2018	2019	2020	Average	Standard Deviation
USD	Safety Cost	1,869,484	1,143,824	286,234	811,732	1,145,695	1,039,192	1,049,360	470,769
USD	Labor cost	1,000,608	1,023,419	1,179,021	583,316	1,095,567	850,748	955,446	193,960
USD	Materials	529,372	349,770	84,216	135,668	441,541	198,217	289,797	162,494
USD	Depreciation	1,491,945	1,408,482	1,681,748	1,724,791	3,101,246	2,824,272	2,038,748	666,858
USD	Others	95,564	64,943	47,940	37,105	661,762	230,894	189,701	220,736
USD	Total	4,986,973	3,990,438	3,279,161	3,292,613	6,445,812	5,143,323	4,523,053	1,128,560
Ton Sn	Production	199	253	75	40	274	383	204	118
USD/ton Sn	COGS	25,116	15,772	98,734	82,315	23,525	13,429	43,149	34,075
USD/ton ore	COGS	1.84	1.48	0.92	2.74	2.21	1.46	1.78	0.59

General Cost

Unit	Year	2015	2016	2017	2018	2019	2020	Average	Standard Deviation
Ton Sn	Ore Tin Production	26,361	24,347	31,035	44,514	82,522	38,235	41,169	19,727
MT	Sales Volume	30,087	26,677	29,914	33,818	64,801	50,235	39,255	13,737
Usd/Ton	Overhead Cost	2,565	2,877	2,461	1,936	1,294	2,440	2,262	514
USD/MT	Smelting and Refining Cost	731	701	668	1,084	857	1,000	840	156
USD/MT	Sales cost	1,587	1,424	1,738	1,689	1,790	1,503	1,622	130