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Three Modes of Classroom Teaching of School Mathematics Relating to Two Conceptions of Mathematics

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二種の数学観に関連している学校数学の授業の三型

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(論文の掲載について)

次頁以下の英文論文は，平成６年８月１６日から２０日まで中華人民共和国の上海，華東師範大学で開催された ICMI-CHINA REGIONAL CONFERENCE ON MATHEMATICAL EDUCATION の Topic Group 4 に提出した論文(口頭発表)である．Topic Group 4 の Leaderであったシンガポールの Ho-Keong Fong 氏による Topic Groue 4 の集約として本論文の要約が下記報告書 (PP.166-167) に Models for Classroom Teaching として掲載されている．

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口頭発表の終了時にSteven Nisbet 氏(Grifith大学) から特大の拍手を頂き，帰り際に自分の考えと共鳴するところがあるから論文を送りたいとのことであった．帰国後,早速教員の資質に関する同氏の論文が送られてきた．

本研究に関して四つの論文・要約が英文で執筆されている．最初のものは Topic Group 4 にアプライするための要旨１頁のものであり，この要旨の提出によってGroup 4 での口頭発表が認められた．次に執筆したものが次頁以下に掲載する本論文であり，数十部のコピーを作成して学会に持参した．口頭発表終了後の学会最終日の全体会場において Ho 氏から２頁程度の要約を送って欲しいとの依頼があり，本論文と同一の表題の２頁の要約も作成されている．最後の要約となったのが上記の Ho 氏によってProceedings に掲載されたものであるが，この方は前記の２頁の要約そのものではなく，Ho 氏によるまとめであり，Minato の論文は，とか，Minato は．．．を論じている，とかと記されている．

Ho 氏による口頭発表の集約は，以前ある所で若干触れたことはあるが，内容は勿論，緩急をつけたパラグラフ構成，掲載順序と行数の割当，発表者名を要約の冒頭だけでなく途中にも出すか否かといった細かな所にまで神経を行き渡らせた実に見事なものである．集約で私の論文が最初に掲載されたのは，Ho 氏自身の論文を最初に掲載し難かったこと，及び当方に対する敬老精神の表出ではなかったかと推測している．Ho 氏とは平成８年７月にICME-8 (セビリア，スペイン)の WG-3 で再会できた．この際の提出論文は ICME-8 の Proceedings での論文の取り扱いをも勘案しながら提示を検討したい．

THREE MODES OF CLASSROOM TEACHING OF SCHOOL MATHEMATICS RELATING TO TWO CONCEPTIONS OF MATHEMATICS

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Until about one hundred and twenty years ago, we Japanese studied 'Wazan', native mathematics of Japan, which was originating mainly in Chinese mathematics and peculiarly developed by some Japanese mathematicians in the 18th and 19th centuries. In the study of Wazan, 'San-ngi' and ' Soroban', two kinds of hand held calculators, which were also originating in China and revised in Japan, were used. Since the first formal Japanese school system was established along with European system, espcially French one, of education in 1872, the students of the schools under the system began to learn 'Yohzan', which means European arithmetic or mathematics, as an important subject. This consciousness on mathematics seems to be almost held in our society (Becker et al., 1990), especially by adults of Japan.

At the time of around 1940, the main schooling system of Japan consisted of six-years of the compulsory elementary school (entering at six-plus years old), five-years of the middle school for academic elite, three-years of the high school, and three-years of the university. In the middle school, arithmetic, algebra, geometry, trigonometry, and a little amount of functions were taught.

After the reformation of the educational system in 1947, at two years after the end of the World War II, nine-years' compulsory schooling was put in force which consisted of six-years schooling in the elementary, and three-years in the lower scondary (junior high) schools. Following these schools, three-years of the upper secondary (senior high) school and four through six-years of the university are provided.

Nowadays in the lower secondary school, mathematics is learned in three (first grade), four (second grade), and four (third grade) periods with optional study where each period is 50 minutes long, in a week. The contents are divided into, but co-related with three areas: numbers and algebraic expressions, geometrical figures, and quantitative relations (JSME, 1990; Becker et al., 1990).

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Mixing many pieces of information on mathemtics and mathematics education in the world and the long history of our attitudes toward society, humanity, science, and also mathematics, Japanese have developed a characteristic style of mathematics education. And now, mathematics education of our country may be noted for its memorizing and immediate recalling of the multiplication table, and high achievement but low attitude. High (92 or more) pecentage of students in the upper secondary school/age cohort is also notable (Robitaille & Garden, 1989).

The Second International Mathematics Study involving many educational systems, one of which was Japan, was performed, and three volumes involving exact situation of mathematics education of Japan were published (Travers, & Westbury,1989; Robitaille, & Garden, 1989; Anderson, Rayan, & Shapiro, 1989). There are other researches into the nature of mathematics teaching of Japan, especially in contrast with American counterparts (Horvath, 1987; Stigler, Lee, & Stevenson, 1987; McKnight et al., 1987; Stigler, 1988; Becker et al., 1990; Reys et al., 1991; Mayer, Tajika, & Stanley, 1991). Due to the reason that researches into mathematics education of Japan were conducted for obtaining some pieces of valuable information on revising mathematics education of own's countries, attitude toward mathematics education of Japan in these research studies seems to be almost favorable, but mathematics education of Japan has really, I think, some difficult problems.

Although there have been heavy pressures of entrance examinations in Japan from Meiji Era, and it has been a steady problem up to the present, more important and crucial factors affecting teaching and learning mathematics have been recently evoked, which are, in my view, relating to the economic and philosophic change of our society, and corresponding teachers' conceptions of mathematics. As Minato (1983) pointed out, academic mathematics and also school mathematics were, in the past, favorable representatives of European civilization, and they seemed to be necessary for student's future success. Now their status may be drastically declining against our developed society, where students think that favorable representatives of modern civilization are not such a cumbersome subject, but TV, video sets, and several types of game machines, which are arranged in their own rooms.

In spite of the fact that the above mentioned change has been undergone in our society, mathematics teaching and also teachers of mathematics have little changed to suit the changing society and students who are not elites, but Tom, Dick, and Betty with normal, average ability. The discrepancies between society

and students, and teaching mathematics for elite as in the old schooling system and that for all in the compulsory, lower secondary school seem to be more and more increasing.

In the report, I intend chiefly to deal with mathematics education of compulsory schools, the elementary and lower secondary (junior high) schools, of Japan, especially the latter, and present historically the mode of the classroom teaching from the late of the nineteenth to the near end of the twentieth centuries. Following these, a required competence of prospective and in-service teachers of mathematics from now on is analyzed.

THREE MODES OF MATHEMATICS CLASSROOM TEACHING

Three modes of mathematics classroom teaching can be historically observed and identified in mathematics education of Japan. They are as follows:

Mode A: Academic Mathematics-Centered, Lecture Mode,
which was strongly supported before the end of the World War II,

Mode B: Transitional, Question and Answer Mode,
which was widely implemented after the end of the World War II until recently or now, and

Mode C: Activity-Oriented, Problem-Solving and Discussion Mode,
which was recently intended and is expected to be held widely.

In Mode A, teaching of mathematics is exclusively constrained by the content, methods and characteristics of the then academic mathematics. In the mode not only psychology of students but also students themselves were absolutely neglected. For example, geometry, as an important subject of the middle school was identical with, or similar to the first three or four chapters of Euclid. In teaching geometry, demonstration depending upon algebraic operations was strictly prohibited, because geometry was seemed to be stemed differently from algebra (Kikuchi, 1897). Mathematics, taught in middle schools was formal and meaningless.

Owing to the difficulty of getting understood in mathematics, and little motivation of learning mathematics in this mode, classroom teaching should be revised into such a mode as enhancing and involving student's activity, and Mode B appeared. This mode, Mode B is characterized as teacher-question and student-answer style. At the beginning of the revision, a teacher appointed a student to request his answer, and then the student answered. The teacher soon estimated the student's answer, and said 'yes' or 'no', or 'right' or

'wrong'. This early stage of Mode B is called 'One question, one answer' (Ichimon-itto) approach in Japan. Although sometimes a student's answer yielded a variety of rich responses of students, and discussion among students and their teacher evoked, it was rare in the early stage of this mode, and if such discussion happened, teachers who were not talented might be anxious.

The changing of the mode was gradual before the end of the World War II, but it was rapidly and widely implemented after the revision of the school system on the extension of compulsory education to the children of age fifteen.

In Mode B, mathematics teacher of Japan have made very much effective and efficient teaching styles, and attained student's high achievement, applying some theories or methods from psychology, for example competition, motivation, small-steps, concrete operation, formative evaluation, and so forth, seemed to be more or less effective. But the higher achievement was attained, the larger unequilibrated state between achievement of and attitude toward mathematics was actualized. This unequilibration is one of the most important problems to be solved today (Minato, submitted; Minato, in preparation).

As the reflection on the above mentioned problem goes on, and the intention of mathematics curriculum is gradually changed from the acquisition of large amount of mathematical knowledge to student's socialization or culturalization, which might mean almost Germany 'Bildung', the main focus on mathematics education was changed from teaching formal mathematics to meaningful activity of student. Then Mode C, Activity-Oriented, Problem-Solving and Discussion mode of mathematics learning, one of the model was presented by Anbo (1968), did or is going to appear.

In Mode C, teachers do not stress teaching but learning. This alternation of the quality of teaching is recommended by the revised program of mathematics education now in force (JSME, 1990). The program also recommended subjctive learning and student's activity in mathematics classroom, especially in the learning of so-called problem situation learning (Kadai gakushyu, in Japanese).

CORRESPONDENCE TO AND DIFFERENCE FROM BEEBY'S MODEL

Researching into educational systems in developing countries, Beeby (1980) contended that educational progress grows inevitably through the following four stages:

I. Dame School Stage,...like dame schools, or pastor's schools in villages,

II. Stage of Formalism,...meaningless contents and formalistic methods,

III. Stage of Transition,...same goals as II, but more effective methods,

IV. Stage of Meaning,...meaningful contents with affective, aesthetic value.

Beeby says that educational stages are yielded differently by the degree of education of teachers and the amount of training they were received. He states that in Stage I teachers are ill educated and ill trained, in Stage II teachers are ill educated but trained, in Stage III which has roughly the same goals as in Stage II, more effective achievement is requested, and teachers are better educated and trained, and in the last stage of Stage IV teachers are well educated and well trained. In Stage I and Stage II, educated teachers were few, therefore students who would be teachers were not sufficiently educated when they were in elementary schools, but teacher training for prospective teachers was possible in some extent. Although his argument focuses on elementary school education, and 'educated' means generally educated in his theory, our developmental history of the middle school through the lower secondary school from Meiji Era shows such correspondence as Stage II to Mode A, Stage III to Mode B, and Stage IV to Mode C in the sense of the quality of teaching. Schooling on Stage I may correspond to that of private school' Terakoya' in Edo Era before the beginning of the formal educational system in Meiji Era.

In spite of the existence of the above mentioned correspondence between two notions of stage and mode, the reason for transition of teaching mode in Japan might be against Beeby's model in the strengthening of teacher education in mathematics. The possibility of the transition from Mode A to Mode B was partly due to the teacher education in mathematics. When the compulsory education was extended to the lower secondary school, almost all the teachers were at least better educated mathematics teachers, a part of whom were educated at normal schools or special courses of them, which were for prospective elementary school teachers, and others were well educated at advanced normal schools or universities. For the duration of Mode A in Japan, mathematics teachers of the middle school were already well educated in mathematics at advanced normal school or university. Therefore formalism of teaching mathematics in Mode A is not due to the ill education of teachers of mathematics in case of Japan. It can be said that well educated teachers in mathematics may be necessary, but it must not be sufficient for transition of higher mode or stage. To know what kind of force to go to a higher mode is might be very important to research into expected transition from Mode B to Mode C.

CHARACTERISTICS OF LEARNING IN THE MODES

For researching into the above mentioned force, analyses of the characteristics of learning mathematics, perception of mathematics by teachers and the status of mathematics teachers are useful. In Mode A, teaching of mathematics is characterized as indoctrination, and learning is strictly passive. Mathematics is percepted as an absolute truth which are external, and does not commit to humanity but Platonic idea, or God. In this mode, teachers of mathematics are authorities, guardians or speakers of absolute Truh, or God.

In Mode B, teachers must apply psychology to classroom for effective teaching. There are activity of student such as answering teacher's question, but perception of mathematics supported in this mode may be the same as in Mode A, and the status of mathematics teachers is unchanged, except that teachers are estimated with their effectiveness and efficacy of their teaching which are represented by students' achievement scores.

Activity of student is centered in Mode C. In the mode, learning is necessarily personal. This does not mean that topics learned are always different from student to student. Students construct their personal knowledge with their experience. In this mode, mathematics is necessarily personal and internal. Contents and methods of teaching mathematics must be fairly free from those of academic mathematics as Bishop (1988) stressed from cultural perspective of mathematics education.

As the status of mathematics itself is altered, the status of teacher of mathematics should be altered. A teacher of mathematics in this mode may not be a terrible speaker of Truth who earnestly says 'right' or 'wrong', or 'true' or 'false' to his students, but a planner, introducer, co-ordinator, co-worker, or supporter for students in mathematics classroom.

TEACHER'S CONCEPTIONS OF MATHEMATICS

There would be mathematicians who believed formal and external conceptions of mathematics, for example Bourbaki (1949). This belief is, as Hardy (1969) wrote, that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations', are simply our notes of our observations. This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards.... . His Platonic view, or an external conception of mathematics may be widely distribueted also over mathe-

matics teachers today (Dossey, 1992). Against external conceptions of mathematics there are internal conceptions or Aristoterian view of mathematics or mathematics education (Dossey, 1992).

If we expect the transition from Mode B to Mode C in mathematics education, teachers of mathematics should bear a corresponding suitable conception for this transition, which should be an internal conception of mathematics. By virture of the analyses performed in the above sections, one of the major and necessary transitional forces from Mode B to Mode C is the teachers' possession of internal conceptions of mathematics, when they are well educated in mathematics. Minato and Hamada (1994) has contended that teachers' external, or Platonic view of mathematics can not ensure the subjective learning, which is almost the same as the learning qualified in Mode C.

CONCLUSIONS

In Japan it has been said teacher's ability is the most important factor for good education, and up to the present teacher's ability expected is almost only the ability in academic mathematics, and in addition, if permitted, earnest for teaching, and positive attitude toward students are desirable. This expectation coincides with Beeby's model, and may not greatly vary from country to country. McGalliard (1983) suggests that teachers do not possess abilities to picture deep and wide goals of mathematics teaching for realizing problem-solving mode, and they are not so educated and not so expected.

The study insists on that the transition of mathematics teaching to Mode C necessitates the teachers' possession of internal conceptions of mathematics. Several previous researches (Ernest, 1985; Thompson, 1984; Brown, 1985; Cooney, 1985; Kesler, 1985; Owens, 1987; Thompson, 1992; Dossey, 1992) suggested that conceptions of mathematics possessed by teachers affected more or less their teaching of mathematics. Thompson (1984) tells us that teachers' belief about mathematics and teaching plays a significant, albeit subtle, role in shaping their behavior. If he had conducted his research bringing a focus on the teaching mode as in this study, he would have found conceptions of mathematics more important. As cited above, Minato and Hamada (1994) concludes that teacher's Platonic view contradicts student's learning of mathematics in Mode C. Conceptions of mathematics will be dealt with as a more and more important factor in mathematics education (Hersh, 1979; Brown, Cooney, & Jones, 1990).

How can we solve the problem of teachers' possession of expected concep-

tions of mathematics? Conceptions of mathematics of a teacher have been formed mainly through the experience he was taught by his teachers. But his teachers were taught by their teachers, and so on. This situation is exactly alike in case of general education for revising teacher's ability in developing countries described by Beeby (1980). In this paradigm change in mathematics education, prospective and in-service education providing some courses of philosophy and history of mathematics, and history of mathematics education as well as curriculum of mathematics should be required.

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